

CONTENTS

Preface	xiii
1. Sets	1
1.1 Preliminaries	1
1.2 Algebra of Sets	4
1.3 Venn Diagrams	9
1.4 Power Set	10
1.5 Countable Sets	11
1.6 Some Special Maps (Functions)	17
1.6.1 The characteristic function	21
1.7 Partitions of Sets	22
1.8 The Minset and Maxset Normal Forms	24
1.9 Multisets	31
2. Propositional Calculus and Logic	41
2.1 Propositions	41
2.2 Compositions of Propositions	43
2.3 Truth Tables and Applications	45
2.4 Some Further Applications of Logic	55
2.5 Functionally Complete Set of Connectives	63
2.6 The Connectives NAND and NOR	64
3. More on Sets	69
3.1 The Principle of Inclusion and Exclusion	69
3.2 The Pigeonhole Principle	81
3.2.1 Some typical applications of the pigeonhole principle	82

3.3	Binary Relations	89
3.3.1	Relations	89
3.3.2	Equivalence relations	89
3.3.3	Union, intersection and inverse of relations . . .	91
3.3.4	Composition of relations	93
3.3.5	The matrix of a relation	95
3.3.6	Closure operations on relations	98
4.	Some Counting Techniques	107
4.1	The Principle of Mathematical Induction	107
4.2	Strong Induction	114
4.3	Arithmetic, Geometric and Arithmetic-Geometric Series	131
4.4	Permutations and Combinations	144
4.4.1	Rules of product and sum	144
4.4.2	Permutations	147
4.4.3	The arrangements of objects that are not all distinct	149
4.4.4	Combinations	152
4.4.5	Generation of permutations and combinations	156
5.	Recurrence Relations	167
5.1	Partial Fractions	167
5.1.1	Rational functions	167
5.1.2	Partial fractions	168
5.1.3	Procedure for resolving into partial fractions	170
5.1.4	Some solved examples	173
5.2	Recurrence Relations: Preliminaries	180
5.2.1	Homogeneous solutions	183
5.2.2	Particular solutions	187
5.2.3	Solution by the method of generating functions	193
5.2.4	Some typical examples	197
5.2.5	Recurrence relations reducible to linear recurrence relations	205

6.	Partially Ordered Sets	213
6.1	Preliminaries	213
6.2	Hasse Diagrams	216
6.3	Chains and Antichains in Posets	221
7.	Graphs	241
7.1	Preliminaries and Graph Terminology	241
7.1.1	Some typical examples	261
7.2	Paths and Circuits	265
7.3	Shortest Path in Weighted Graphs	271
7.4	Eulerian Paths and Circuits	281
7.5	Hamiltonian Paths and Circuits	298
7.6	Planar Graphs	309
7.6.1	Applications	313
7.6.2	Some further examples	316
7.6.3	Graph colouring	319
7.7	Matrix Representations of Graphs	325
7.7.1	Adjacency matrix	325
8.	Trees	343
8.1	Introduction and Elementary Properties	343
8.2	Rooted Trees	350
8.3	Tree Searching or Traversing a Tree	362
8.4	Applications of Trees	376
8.4.1	Prefix codes	376
8.4.2	Binary search trees	384
8.4.3	On counting trees	388
8.4.4	Some further examples	395
8.5	Spanning Trees and Cut-Sets	400
8.6	Minimal/Minimum/Shortest Spanning Tree	416
9.	Groups	443
9.1	Groups: Preliminaries	443
9.2	Subgroups	449
9.2.1	Lagrange's theorem	451
9.3	Quotient Groups	455
9.4	Symmetric Groups	457

10.	Rings	467
10.1	Rings	467
10.2	Polynomial Rings	470
10.3	Quotient Rings and Homomorphisms	474
11.	Fields and Vector Spaces	481
11.1	Fields	481
11.1.1	Field extensions and minimal polynomial	484
11.1.2	Characteristic of a field	485
11.1.3	Splitting field	485
11.2	Vector Spaces	491
11.2.1	Basis of a vector space	494
11.2.2	Subspaces and quotient spaces	498
11.2.3	Linear transformations	504
12.	Lattices and Boolean Algebra	509
12.1	Lattices	509
12.2	Lattices as Algebraic Systems	515
12.3	Sublattices and Homomorphisms	521
12.4	Distributive and Modular Lattices	525
12.5	Complemented Lattices	541
12.6	Boolean Algebras	545
12.7	Boolean Polynomials and Boolean Functions	554
12.8	Switching (or Logical) Circuits	567
13.	Matrices, Systems of Linear Equations and Eigen Values	577
13.1	Linear System of Equations	577
13.1.1	Rank of a matrix	577
13.1.2	Linear system of equations	578
13.2	Elementary Row Operations, Gaussian Elimination	581
13.2.1	Elementary row operations	581
13.2.2	Gaussian elimination in matrix form	583
13.2.3	Gaussian elimination method	585
13.2.4	Direct methods for the solution of linear system of equations	591

13.2.5	Method of factorization	594
13.2.6	Some additional examples	598
13.3	Eigen Values	600
13.3.1	Eigen values and eigen vectors	603
	Bibliography	613
	Index	615