Preface

The use of special functions, and in particular of Airy functions, is rather common in physics. The reason may be found in the need to express a physical phenomenon in terms of an effective and comprehensive analytical form for the whole scientific community. However, for almost the last twenty years, many physical problems have been solved by computers, a trend which is now becoming the norm as the importance of computers continues to grow. Indeed as a last resort, the special functions employed in physics will have to be calculated numerically, even if the analytic formulation of physics is of prime importance.

Airy functions have periodically been the subject of many review articles, most of which focus on tabulations for the numerical calculation of these functions, as this is particularly difficult. The most well-known publications in this field are the tables by J. C. P. Miller, dating from 1946, and the chapter in the *Handbook of Mathematical Functions* by Abramowitz and Stegun, first published in 1954. Since that time, no noteworthy compilation of Airy functions has been published[1] in particular about the calculus involved in these functions. For example, in the latest editions of the tables by Gradshteyn and Ryzhik, they are hardly mentioned. Similarly, many results that have accrued over time in the scientific literature, remain extremely scattered and fragmentary.

Although Airy functions are used in many fields of physics, the analytical outcomes that have been obtained are not (or only poorly) transmitted among the various fields of research, which basically remain isolated. Moreover the tables by Abramowitz and Stegun are still the only common

[1]Note however that recently an equivalent compilation has become available on the web: the NIST Digital Library of Mathematical Functions – Chap. 9 Airy & Related Functions by F. W. J. Olver. Internet address: http://dlmf.nist.gov/

reference for all the authors using these functions. Thus many of the results have been rediscovered, and sometimes extremely old findings are the subject of publications, which represent a wasted effort for researchers.

The aim of this work is to make a rather exhaustive compilation of current knowledge on the analytical properties of Airy functions. In particular, the calculus involved in the Airy functions is carefully developed. This is, in fact, one of the major objectives of this book. While we are aware of repeating previous compilations to a large extent, this seemed necessary to ensure coherence. This book is addressed mainly to physicists (from advanced undergraduate students to researchers). We make no claim about the rigour of the mathematical demonstrations, as the reader will see.[2] The main aim is the result, or the fastest way to reach it. Finally, in the second part of this work, the reader will find some applications to various fields of physics. These examples are not exhaustive; they are only given to succinctly illustrate the use of Airy functions in classical or in quantum physics. For instance, we point out to the physicist in fluid mechanics that he can find what he is looking for in works on molecular physics or surface physics, and *vice versa.*

O. Vallée *&* M. Soares, Fall 2003

Preface to the second edition. In this second edition we have corrected various misprints of the first edition. Moreover we have added, here and there, new material such as catastrophe theory for the generalisation of the Airy function, additional results concerning the Airy transform and applications for instance to the Airy kernel, *etc.* It is our hope that this new edition will keep the book up to date in this still useful field.

We would like to thank Pr. Vladimir Varlamov and Pr. Sir Michael Berry for pointing out several omissions and flaws in the first edition.

O. Vallée *&* M. Soares, January 2010

[2] As a matter of fact, the Airy function can be considered as a distribution (generalised function) whose Fourier transform is an imaginary exponential. Also most of the integrals evoked in this work should be evaluated with the help of a convergence factor.