1

Physics of the Universe

Introduction

Physics is the science of inanimate matter. Cosmology is the part of this science that deals with the universe as a whole. It is the oldest and the youngest branch of physics. It is the oldest because the heavens were studied in ancient times, in Greece and in Asia, and other parts of the world. It is the youngest because it has been re-invigorated in recent times due to observations with new, high-resolution astronomical instrumentation (such as the Hubble telescope) and theoretical analyses in the context of current thinking in particle physics and relativistic dynamics. Voluminous works have been written on the order of the night sky. (*The Greek word, 'Cosmology', means 'order'* (*logos*) *of the cosmos*.)[1]

Astronomical laboratories have been constructed since the ancient times to study this order. Examples include the Stonehenge monument, built by the ancient Britons thousands of years ago, and similar ancient astronomical viewing sites in India, China, Australia, Peru, Mexico and from other cultures in the different corners of the world, designed by the ancient and aboriginal peoples to see the star formations and their locations, the locations of the sun and the moon, at the different times of the year. In these ancient viewings, there was no magnification.

Galileo, in the 16th century, was the first astronomer to use magnification, utilizing the telescope — a series of lenses that he contrived to view the heavens.[2] Focusing mainly on our solar system, he saw the moons of Jupiter, the sun spots, the landscape of the moon, and he verified the conclusion of Copernicus that *the earth moves*! However, Galileo went further than Copernicus, who theorized that the sun is at the absolute center of the universe, and that the earth orbits about it, along with the other planets of the solar system. Galileo said there is no absolute center of the universe and that 'motion' *per se* is not an objective quality of matter.[3] Rather, it is a subjective element in its description. Thus, in Galileo's view, it is just as true to say, *from the earth's perspective*, (i.e. its frame of reference) that the sun rotates about the earth, as it is to say that, *from the sun's perspective*, the earth rotates about the sun. Indeed, it was Galileo's view that the laws of nature underlying the binding of the earth to the sun (and vice versa) are independent of the perspectives taken from which the observations of their binding ensues. This is called *Galileo's principle of relativity*.[4] It is an important precursor for '*Einstein's principle of relativity*' that underlies his theory of relativity.

In the generation that followed Galileo, Newton (who was born the same year as Galileo died, 1642) discovered the laws of motion of things (discrete masses) and the law of universal gravitation. He also perfected the means of viewing the night sky with a new, higher-resolution type of telescope — a 'reflecting telescope' — that was a collection of mirrors rather than lenses, much smaller in dimension than Galileo's lens-type telescope. Newton also did research in his attempt to understand optical phenomena. His view was a mechanistic one, where light is assumed to be a collection of particles. (*Newton's contemporaries, Hooke and Huygens, believed in a continuous wave theory of light. It was later verified that Newton's corpuscular model of light was wrong, that light is really a continuous wave phenomenon, whose underlying basis is electromagnetic radiation*.)[5]

In the 18th century, William Herschel discovered that the 'Milky Way' is not the entire universe, as Galileo believed. Herschel found that our galaxy, 'Milky Way', has a neighboring galaxy of stars, called 'Andromeda'.[6] This was discovered later on to be a member of a binary system with 'Milky Way'. Later in the history of astronomy, it was found that the universe consists of an indefinitely large number of galaxies, each containing an indefinitely large number of stars. The galaxies are not distributed in the universe homogeneously and isotropically. Rather, they cluster in certain regions and are absent in others.

With the new high-resolution instrumentation, it was found in the 20th century that most of the galaxies are pancake-shaped, with spiral arms, where the highest density of stars is toward their centers. Our sun is an average-sized star residing in one of the spiral arms of the 'Milky Way'. We also know that the galaxies rotate about an axis that is perpendicular to their planes. Some of the galaxies that do not have spiral arms have the (egg-like) shapes of ellipsoids. It is possible these are in an evolutionary stage, later to develop spiral arms. It was found that the rotations of the galaxies cannot be attributed to the Newtonian gravitational pull of the neighboring galaxies. They would not have the sufficient magnitude. It has been proposed that there is unseen *dark matter* in the regions of the galaxies (and throughout the universe) that is responsible for their rotations. The details of dark matter will be discussed in Chapter 5 (pp. 51–59).

Newton's equations of motion and his theory of universal gravitation imply all of the heavenly bodies are in stationary orbits about other bodies, just as the planets of our solar system, *from the perspective of the sun*, are in stationary orbits about the sun. This is because of the separation of the (relative) functional dependence on spatial coordinates from the functional dependence on the (absolute) time coordinates in Newton's equations of motion. The equation of motion in the spatial coordinates, in Newton's theory, predicts the elliptical orbits and constant angular momenta of a rotating body, such as the earth, relative to its center of rotation, the sun, whose center of mass resides at one of the elliptical foci. This confirms Kepler's discovery, a generation before Newton, that the orbit of Mars is elliptical about the sun, which is at one of the elliptical foci, and his generalization of this observation to a law of orbital motion of all the planets.

Is Newton's Theory an Explanation of Gravity?

One feature of Newton's theory that puzzled him was 'action at a distance'. That is, a body spontaneously acts on another body, irrespective of the magnitude of their spatial separation. He said the following about this concept: '*Action at a distance through a vacuum, without mediation of anything else by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man who has in philosophical matters competent faculty of thinking, can ever fall into it*'. He said this in a letter to R. Bentley III, in 1693. [See A. Janiak (ed.); *Newton*: *Philosophical Writings* (Cambridge, 2004), p. 102. Also see M.B. Hesse, *Forces and Fields* (Nelson, 1961), p. 152]. But he excused his use of the concept by saying: 'because it works! I do not form hypotheses'. Yet, from a study of his writings, it is clear that Newton did indeed form hypotheses in physics! Because of his unhappiness with 'action at a distance', I believe Newton did not yet think that he had *explained* gravitation, though he *described* it adequately for his time.

It was not until the fruition of Einstein's theory of general relativity, three centuries later, that gravitation was explained satisfactorily. Two features that differed in Newton's versus Einstein's theories of gravitation were: (1) atomism versus the continuous field concept and (2) action at a distance versus the propagation of forces at a finite speed. A further difference was that in Newton's theory, the space parameters and the time parameters are separate from each other in the sense that the time measure is absolute — it does not change in the expression of a law of nature from one reference frame to another, but a spatial measure in one reference frame is transformed to different spatial measures in other reference frames where the law of nature is expressed. But in the theory of relativity, a spatial (or a time) measure is transformed to a mixture of a spatial and a time measure in other reference frames where the laws of nature are expressed. The time measure then becomes relative to the reference frame in which the law of nature is expressed. This is in contrast to Newton's classical theory, where the time measure is absolute, i.e. frame-independent. Thus, in relativity theory, space and time become *spacetime*. Such a language for the laws of nature then does not predict stationary orbits for the heavenly bodies.

The Expanding Universe

The lack of stationary orbits of the heavenly bodies of the universe was verified by Hubble in his discovery of the expanding universe.[7] This observation was in agreement with the prediction of no stationary orbits of matter of the universe, according to Einstein's theory. In our local domain of the solar system, at first glance there seems to be the stationary orbits of the planets relative to the sun, but Einstein's theory showed that these are really not stationary either, as exemplified in the observation of the perihelion precession of Mercury's orbit (i.e. the planet Mercury does not return to the same place relative to the sun, after each cycle of its orbital motion). Hubble's data revealed the fact that the galaxies of the universe are moving away from each other at an accelerating rate, in accordance with the Hubble law: $v = Hr$, where v is the speed of one galaxy relative to another and r is their mutual separation. This cosmological dynamics is referred to as 'the expanding universe'.

This dynamics does not mean to imply that the matter of the universe, as a whole, is expanding into empty space! For there is no empty space as a 'thing in itself'. The universe is, by definition, all there is. What is meant, physically, by the 'expansion' of the universe is that from any observer's view, the density of matter, anywhere in the universe, is decreasing with respect to his or her time measure.

The way that Hubble detected this expansion was to measure the Doppler Effect of the radiation emitted by the galaxies. If a source of radiation emission is moving away from an absorber of this radiation, its absorbed frequency components will shift toward the longer wavelength end of the spectrum — this is a 'red shift' for the visible spectrum. Thus, Hubble saw the spectrum from a more distant galaxy red shifted from a similar spectrum of a closer galaxy. This would mean that the density of matter in the observed stellar domain of the universe is decreasing in time, according to any particular observer's measurements.

If the universe is indeed monotonically expanding in this way, then if one should extrapolate to the past, the universe was increasingly dense at the earlier times. The extrapolation would then reach a limit, where the matter of the universe was maximally dense and unstable. At that point in time, according to any observer's view, the universe would have exploded, starting off the presently observed expansion. This event of the universe is commonly referred to as the 'big bang'.[8]

The question then arises: how did the matter of the universe get into this state of maximum density and instability in the first place? This is a scientific question and requires a scientific answer. To answer this question theologically by saying that this point in time was when God created the universe is a nonsequitur. It is based on religious truth, founded on irrefutable faith. That is not to say religious truth is false. Rather, it is in a different context to scientific truth. Scientific truth is in principle refutable and based on scientifically testable empirical facts and logical consistency of the basis of this theory. This difference will be discussed further in Chapter 7 (pp. 64–73).

The Oscillating Universe Cosmology

The only *scientific* answer to this question that I see is this: before the big bang event and the subsequent expansion phase of the universe, where the dominant gravitational force between galaxies was repulsive, the universe was imploding with a dominant attractive gravitational force between the matter components, contracting as a whole toward ever-increasing density. An inflection point was then reached, where the dominant attractive forces changed to dominant repulsive forces, and the contraction changed to an expansion. Eventually, when the density of the matter of the presently expanding universe will become sufficiently rarefied, another inflection point will be reached, where the dominant repulsive forces between the matter components of the expanding universe will change to a dominant attractive force. Once again, the expansion will then change to a contraction and the matter of the universe will implode until the next 'big bang', initiating the next expansion phase. The dynamics of the universe as a whole continue in cyclic fashion between expansion and contraction. This is compatible with the meaning of time in relativity theory as a non-absolute measure. That is to say, this model in cosmology rules out the concept of an absolute temporal *beginning* of the universe, from a scientific stand.

The oscillating universe cosmology follows in physics from the theory of general relativity. The terms that play the role of force in this theory depend on the geometrical functions of the curved spacetime, called 'affine connection'.[9] They are not positive-definite. Thus, under one set of physical conditions, when matter is sufficiently dense and the relative speeds of matter are close to the speed of light, the gravitational forces are repulsive, predicting that matter moves away from other matter. But when the matter becomes sufficiently rarefied in the expansion phase, the dominant repulsive forces between matter become dominantly attractive, leading to the contraction of the matter of the universe. (In the latter phase of the oscillating universe, the *redshift* of radiation observed by Hubble, implying an expansion would change to a *blueshift* — a shift of the visible spectrum toward the shorter wavelengths, as observed when the source of the radiation is approaching the absorber of this radiation, rather than receding from the absorber. This would imply a contracting universe.)

Thus, with this scenario, the universe is continually expanding and contracting — there were continual 'big bangs' in the indefinite past and will continue in the indefinite future. The time of the last 'big bang', estimated (from the Hubble law) to be about 15 billion years ago, was only the beginning of this particular cycle of the oscillating universe.

The Theory of General Relativity

The underlying physical dynamics of the universe as a whole is given by Einstein's theory of general relativity. This theory is based on a single axiom: 'The principle of relativity' (also known as the 'principle of covariance'). It is the assertion that any law of nature for any particular phenomenon, as expressed by any observer, with respect to a comparison in all possible reference frames relative to his or her own reference frame, must be in *one-to-one correspondence*. This is equivalent to saying that the laws of nature must be totally objective — independent of reference frames.

It might be argued that the theory of relativity, according to this definition, is a tautology rather than a science. For how could a law be a law, by definition, if it were not fully objective? It would be like saying: a human female is a woman. Of course this is a true statement, but it is only the definition of a word. (*It is called a 'necessary truth'*.) Nevertheless, the principle of relativity is not a tautology because it depends on two implicit assumptions that are not tautological. One is the idea that there are laws of nature in the first place. The second implicit assumption is that we can comprehend and express the laws of nature. These are not *necessary truths*. They are *contingent*, and thus qualify as scientific statements. (*This view supports Karl Popper's distinction between a necessary truth and a scientific truth*.[10])

The assertion of the existence of laws of nature corresponds to saying that for every physical effect in the world, there is an underlying physical cause. Here, there is an implication that the universe is totally ordered. It is the scientist's obligation, then, to pursue this order in terms of the cause-effect relations — the laws of nature. If, as Galileo believed, it is impossible to achieve a *complete knowledge* of the order of the universe,[11] regarding the laws for any of its phenomena, it is still the obligation of *scientists* to pursue an *increase* in their knowledge and understanding, though never expecting to reach complete comprehension.

The Role of Space and Time

The second implicit assumption is that we can comprehend and express the laws of nature, in our own language. It is the latter assumption where the space and time measures come in. They are the 'words' of a language that we invent (not the only possible language!) for facilitating an expression of the laws of nature.

It is important to recognize that the laws of nature, *per se*, are not identical with the language that we invent to express them. Nature is there, it is *of* the universe, whether or not we are there to express its characteristics. Our language, on the other hand, is invented by us, to help us express the laws of nature. Space and time are not physical things that can contract, distort, and so on. Thus, it is not correct to say that the existence of matter distorts spacetime, as a lead sphere may distort a rubber sheet that it falls into! Space and time are not more than the elements of a language we invent for the purpose of facilitating an expression of the laws of nature — such as the physical laws of the universe. This is a mathematical language that is continually perfected, perhaps for want of a better mathematical language to express the laws of nature.[12]

As we have discussed earlier, three space parameters and the time parameter in the laws of nature, according to relativity theory, do not have any objective (i.e. frame-independent) significance in themselves. What is significant as an objective language is the unification of space and time into spacetime. This means that purely space measures in the expression of a law of nature in one reference frame is a combination of space and time parameters in the expressions of the same law of nature in different reference frames. It then follows that the time measure must be expressed in the same units as the space measure. That is, in the different reference frames where the laws of nature are compared, instead of calling the time measure t seconds, t' seconds, t'' seconds. etc., they must be called ct centimeter, ct' centimeter, ct'' centimeter, etc. where c, as a conversion factor, must be frame-independent, with the dimension of centimeter/second — the dimension of a speed. Thus, the principle of relativity predicts there must be an invariant speed associated with the time measure. It turns out, in looking at one particular law of nature — the Maxwell field equations that underlie electromagnetism — that c is the invariant speed of light in a vacuum. (*In the initial stages of relativity theory, Einstein said that there were two independent axioms that underlie this theory*: (1) *the principle of relativity and* (2) *the invariance of the speed of* *light. We see here that the latter is not an independent axiom — it follows logically from the principle of relativity*.)

The spacetime language forms a continuous set. The principle of relativity then requires that the transformations of the expressions of the laws of nature, such as the laws of cosmology, in one reference frame, to a *continuously connected* reference frame, must leave the form of the law unchanged. (It is called 'covariance'.) This requirement implies that the laws themselves must be *continuous* field equations (as is the Maxwell formulation for electromagnetism) and that the solutions of these laws are *continuous fields* (everywhere). One further requirement of the transformations of relativity theory, according to the principle of relativity, is that discovered by Emma Noether, called *'Noether's theorem'*.[13] It is that a necessary and sufficient condition for the laws of conservation (of energy, momentum and angular momentum) to be included with the other laws of nature, in the special relativity limit of the theory, is that the transformations must not only be continuous, but also analytic. That is, their derivatives must exist to all orders. Singularities are then automatically excluded. The set of continuous, analytic transformations, everywhere, forms a *Lie group*. In the case of the theory of general relativity, this is the 'Einstein group'. In the special relativity limit of this theory, this is the 'Poincaré group'. These Lie groups underlie the algebraic logic of the theory of relativity. The functions that transform in this way, to maintain the principle of relativity, are called 'regular', indicating the inadmissibility of singularities anywhere in the universe! This would include a rejection of the 'black hole' (as it is commonly understood in physics today), and the singularity of the 'big bang' in cosmology. It is a condition on the solutions of general relativity, emphasized by Einstein throughout his lifelong pursuit of this theory. That is, Einstein would not have accepted either of these singularities, commonly discussed today, as realities![14] (*In personal discussions with Prof. Nathan Rosen, one of Einstein's close collaborators in the 1930s, he acknowledged the validity of this statement*.)

The Lie group of the theory of general relativity, the Einstein group, has 16 essential parameters; these are the derivatives of the four spacetime coordinates of one (primed) reference frame where the laws are expressed, with respect to those of a different (unprimed) spacetime reference frame, $\partial x^{\mu'}/\partial x^{\nu}$, where μ and ν refer to the four space and time coordinates in each reference frame. The significance of the number 16 of essential parameters of the Lie group of general relativity theory is that there must be 16 independent field equations to prescribe the spacetime.[15] What is interesting here is that Einstein already showed that there are (at least) 10 independent equations (the 'Einstein field equations'), with 10 solutions, that underlie the gravitational phenomenon, and there are 6 independent solutions of the laws of electromagnetism — the three components of the electric field and the three components of the magnetic field. It has been shown that the Lie group of general relativity then implies a truly unified field theory, with a 16-component metrical field, where gravity and electromagnetism are unified in terms of a single field of force. This new 16-component metrical field of general relativity theory is a four-vector field $q^{\mu}(x)$, in which each of the four components is a quaternion rather than a real number field.[16] *The new quaternion formulation of general relativity theory implies a cosmology in which the expansions and contractions of the oscillating model of the universe, as a whole, are spiral rather than isotropic, and where the Hubble law is predicted as a first approximation. This is discussed in Chapter* 4 *and in Ref.* 17.

Geometry and Matter

Einstein started his theory of general relativity with the idea that the variability of the matter content of the universe implies a variability of the coefficients of the metric tensor that underlies the geometry in the language of spacetime. This is tied to the idea that the spacetime is not more than a language that *reflects* the physical properties of the matter content of the universe. Analogous to the logic of ordinary verbal language, this language, in turn, has a geometrical logic and an algebraic logic. The algebraic logic is in terms of the underlying symmetry group of the theory, as discussed above. The geometrical logic is expressed in terms of the invariant differential metric of the spacetime:

$$ds^2 = g^{\mu\nu}(x)dx_\mu dx_\nu = ds'^2,$$

where summation is implied over the subscripts and superscripts, and $\mu, \nu = 0, 1, 2, 3$ are the temporal (0) and spatial $(1, 2, 3)$ coordinates. $g^{\mu\nu}(x)$ is the 'metric tensor'. It is, in Einstein's nonsingular field theory, a 'regular function', *everywhere*, and a second-rank, symmetric, 10-component tensor. Its continuous variability in the spacetime x is a *reflection* of the continuous variability of the matter fields of the spacetime. The idea is then that the spacetime transformations that keep the invariance of $ds^2 = ds'^2$ are those that transform *any* law of nature covariantly — i.e. preserving its form in all (continuously connected) reference frames. This is a *Riemannian geometrical system*.

The geodesic of the spacetime — the path of minimum separation between any two of its points — is determined by minimizing the invariant integral that is the path length $\int ds$. The physical significance of the geodesic is that it is the natural path of an unobstructed body. In Galileo's classical view, the geodesic is a straight line; the family of such straight lines is a 'flat spacetime'. *Galileo's principle of inertia* then asserts the natural path of an unobstructed body is a straight line. In contrast, the geodesic of the Riemannian spacetime is a curve. Thus, the natural path of an unobstructed body in such a spacetime is a curve. The family of such curves is a 'curved spacetime'. The variables of this curve, that gives it its structure, are determined by the variability of the matter of the spacetime. As the matter of the system is then continuously depleted, the curved spacetime approaches a flat spacetime. In this limit of a perfect vacuum, *everywhere*, the values of the metric tensor become:

$$g^{00} \to 1, \quad g^{kk} \to -1, \quad g^{\mu\neq\nu} \to 0, \quad (k = 1, 2, 3)$$

so that, for a perfect vacuum, *everywhere*, the invariant metric becomes:

$$ds^2 = (dx_0)^2 - (dx_1)^2 - (dx_2)^2 - (dx_3)^2 = ds'^2$$

This is indeed the invariant metric of special relativity theory — the 'Lorentz metric'. Thus, the theory of special relativity, discovered originally to preserve the forms of the laws of nature in inertial fames of reference (i.e. frames in constant, rectilinear speed relative to each other) really only applies to the ideal case of the perfect vacuum. Where its formalism 'works' in physics, for a material medium (e.g. in its prediction of the energy- mass relation, $E = mc^2$, the dynamics of elementary particles, etc.) must then be a mathematical approximation for the metric of general relativity. This assumes the curvature of spacetime, in a small region, may be *approximated* by a flat spacetime, tangent to the curved spacetime where it is applied.

The idea of the theory of general relativity, as discussed earlier, is that the geometry of spacetime is to *reflect* the matter content of the closed system, in principle the universe. For example, the existence of the sun is *reflected* in the curved spacetime geometry in the vicinity of the sun, causing the trajectory of a beam of starlight to bend as it passes this region of space. This bending was predicted qualitatively and quantitatively by Einstein and it was then observed in agreement with his theory of general relativity. It is a gravitational effect that was not predicted by the earlier Newtonian theory of universal gravitation. Along with other gravitational effects not predicted by the classical theory, as well as giving back the equations of the classical theory *as an approximation*, Einstein's theory of general relativity superseded Newton's theory of universal gravitation, as a true *explanation* of gravity. Thus, Einstein discovered the field equations that predict the phenomenon of gravity. It relates the geometry of spacetime, in terms of the Riemannian metric tensor, $g^{\mu\nu}(x)$ and its changes in spacetime to the matter field variables of the closed system, chosen to be the energy-momentum tensor of its material content.

Generalization of Einstein's Field Equations

Einstein's field equations, in $g^{\mu\nu}(x)$, are 10 independent, nonlinear differential equations. But they are too few in number, for this reason: the symmetry group of general relativity theory — the 'Einstein group' — is the group of transformations that defines the invariance of $ds^2 = g^{\mu\nu}dx_\mu dx_\nu = ds'^2$. This is a *Lie group*, a group of continuous, analytic transformations that preserves the forms of the laws of nature under the changes from one frame of reference to any continuously connected frame of reference. This preservation is the requirement of the underlying principle of relativity. The number of *essential parameters* of the Lie group, $\partial x^{\mu'}/\partial x^\nu$, is 16 in number. It implies that the most general form of the field equations, subject to the principle of relativity, must be 16 in number rather than 10.

Why are Einstein's equations 10 in number rather than 16? It is because the form of these equations is more symmetric than they need be, in accordance with the (16-parameter) Einstein group. They are covariant (form-invariant) with respect to the continuous transformations, as they must be. But they are also covariant with respect to the discrete reflections in space and time, which is not required. By lifting the space and time reflection transformations, Einstein's equations thereby *factorize* to 16 independent equations, as it is shown in Chapter 3 (pp. 27–36) and in Ref. 18.

It is important to note that the invariant differential metric of the spacetime is not ds^2, it is ds. How does one take the square root of ds^2? It is usually stated that its square root is $\pm ds$, and the minus sign is simply thrown away! But this is not valid, as the square root is double-valued at all points of the spacetime, i.e. there is an ambiguity in the sign of this term, *everywhere*.

The answer to this question comes from the fact that the irreducible representations of the Einstein group of general relativity obey the algebra of quaternions (as well as the irreducible representations of the Poincaré group, of special relativity). The quaternion is a generalization of the complex number in 2-dimensional space, whose basis elements are 1 and i ($= \sqrt{-1}$), forming the complex function, $f(z = x + iy) = u(x, y) + iv(x, y)$.[19] The basis elements $(1, i)$ of the complex number in two dimensions generalize to the four basis elements, σ^0, σ^k, (where $k = 1, 2, 3$); these are the unit two dimensional matrix and the three Pauli matrices. Thus, the quaternion has the *form* in the 4-dimensional space

$$q(x) = \sigma^0 x_0 + \sigma^1 x_1 + \sigma^2 x_2 + \sigma^3 x_3$$

We then define the quaternion four-vector $q^{\mu}(x)$, with the differential invariant quaternion of general relativity,

$$ds = q^{\mu} dx_{\mu}$$

Thus, there is no ambiguity here as to the sign of the invariant ds. It is important, and clear from this form of the quaternion, in terms of two-dimensional matrices that the product of two quaternions is not commutative under multiplication, i.e. $q_1 q_2 \neq q_2 q_1$.

Each of the four components of the four-vector q^{μ} is quaternion-valued and thus has four components. Thus, this quaternion metrical field has $4 \times 4 = 16$ components. This is a unique expression for the invariant $ds = q^{\mu} dx_{\mu}$. Further, the quaternion metric field transforms as a second rank spinor, that is, q^{μ} transforms as $(\psi \times \psi^*)^{\mu}$. Thus, the basis elements of the quaternion are two-component spinor variables ψ.

We see here that any law of nature, *whether in particle physics or in cosmology — the physics of the universe* — that is compatible with the symmetry required by relativity theory, in special or general relativity, must be in terms of spinor and quaternion variables. This is a requirement of the algebraic logic — the group structure — of the theory of relativity. It is the reason why Dirac's special relativistic theory of wave mechanics led to spinor degrees of freedom in the description of the electron (and a quaternion operator to determine these solutions). That is, the spin degrees of freedom in Dirac's electron equation are not a consequence of quantum mechanics, *per se*, as many have claimed! It is a consequence of the symmetry imposed by the theory of relativity.

The correspondence of the quaternion metric field and the metric tensor of Einstein's formulation is then in terms of the product of ds and its quaternion conjugate ds^* (corresponding to its time (or space) reflection):

$$ds\, ds^* \approx -(1/2)(q^{\mu} q^{\nu *} + q^{\nu} q^{\mu *}) dx_{\mu} dx_{\nu} = g^{\mu\nu} dx_{\mu} dx_{\nu}$$

Thus, the quaternion formulation in general relativity, $ds = q^{\mu} dx_{\mu}$, is a factorization of the metric tensor formulation of the standard Einstein theory.

Similar to Einstein's derivation of the field equations in $g^{\mu\nu}$ from a variational principle, the factorized field equations in q^{μ} may be derived from a variational principle. These derivations are shown in Ref. 16. The quaternion form yields 16 field relations that replace the 10 relations of Einstein's symmetric tensor form of his field equations in general relativity. In addition to explaining gravity, as will be described in Chapter 3 (pp. 27–36), the quaternion form predicts new physical effects in the cosmological problem of the universe as a whole. One important new feature is the torsion of spacetime. It predicts, as examples, the rotation of the galaxies and the Faraday Effect regarding the propagation of cosmic electromagnetic radiation, i.e. the rotation of the plane of polarization of this radiation as it propagates throughout the universe. Both are observed astrophysical effects, not predicted by the standard Einstein tensor formulation. A further prediction is an anisotropic expansion and contraction of the universe, in a spiral fashion. Another important difference is that the geodesic equation, that prescribes a natural motion along a curve of an unobstructed body, has a quaternion form. That is, to prescribe the motion of a body along a trajectory, parameterized by the time measure, one must have four parameters, rather than one, to prescribe the time change, as a body moves from one spatial location along its trajectory to another. This is the generalization of the time parameter in physics theorized by William Hamilton, from his discovery of the quaternion algebra in the 19th century.

A Unified Field Theory

By iterating the 16 field equations in q^{μ} with the conjugated solution $q^{\nu *}$ on the left, and iterating the conjugated (i.e. reflected) equation in $q^{\nu *}$ on the right with q^{μ}, second-rank tensor equations are generated. Adding these two iterated equations generates a symmetric second-rank tensor equation (10 components) that we will see is in one-to-one correspondence with Einstein's original tensor equations (Chapter 3), thus explaining gravity. Subtracting these equations generates an antisymmetric second-rank tensor equation that can then be put into one-to-one correspondence with Maxwell's equations, thus explaining electromagnetism. Thus, the original 16 quaternion metrical field equations break up into 10 equations that explain gravity and 6 equations that explain electromagnetism — *this is the unified field theory that was sought by Einstein*. In this field theory, in general relativity both physical phenomena are incorporated in the single, 16-component quaternion field q^{μ}. It is this formal, generalized expression of general relativity that generates a new cosmology,[17] relating to the physics of the universe. The dynamics is an oscillating universe, between expansions and contractions, in spiral configuration. As will be discussed in Chapter 4 (pp. 37–50), the Hubble law is an approximation for this dynamics, over sufficiently short times in the expansion phases of the universe.

In the next chapter, we will discuss the physics and outline the formal development of Einstein's tensor form of general relativity theory as a language of cosmology.