

# Contents

<i>Preface</i>	vii
<i>Leonhard Euler (1707-1783): Chronology</i>	xiii
1. Mathematics Before Leonhard Euler	1
1.1 Introduction . . . . .	1
1.2 Pythagoras, the Pythagorean School and Euclid . . . . .	2
1.3 The Major Impact of the European Renaissance on Mathematics and Science . . . . .	11
1.4 The Discovery of Calculus by Newton and Leibniz . . . . .	21
2. Brief Biographical Sketch and Career of Leonhard Euler	31
2.1 Euler's Early Life . . . . .	31
2.2 Euler's Professional Career . . . . .	33
3. Euler's Contributions to Number Theory and Algebra	57
3.1 Introduction . . . . .	57
3.2 Euler's Phi Function and Cryptography . . . . .	57
3.3 Euler's Other Work on Number Theory . . . . .	60
3.4 Euler and Partitions of Numbers . . . . .	68
3.5 Euler's Contributions to Continued Fractions . . . . .	78
3.6 Euler's Contributions to Classical Algebra . . . . .	83
4. Euler's Contributions to Geometry and Spherical Trigonometry	101
4.1 Introduction . . . . .	101
4.2 Euler's Work in Plane Geometry . . . . .	102

4.3	Incircle, Incenter and Heron's Formula for an Area of a Triangle . . . . .	112
4.4	Centroid, Orthocenter and Circumcenter . . . . .	115
4.5	The Euler Line and the Euler Nine-Point Circle . . . . .	121
4.6	Euler's Work on Analytic Geometry . . . . .	126
4.7	Euler's Work on Differential Geometry . . . . .	132
4.8	Spherical Trigonometry . . . . .	146
5.	Euler's Formula for Polyhedra, Topology and Graph Theory	153
5.1	Euler's Formula for Polyhedra . . . . .	153
5.2	Graphs and Networks . . . . .	164
6.	Euler's Contributions to Calculus and Analysis	175
6.1	Introduction . . . . .	175
6.2	Euler's Work on Calculus . . . . .	177
6.3	Euler and Elliptic Integrals . . . . .	184
7.	Euler's Contributions to the Infinite Series and the Zeta Function	197
7.1	Introduction . . . . .	197
7.2	Euler and the Infinite Series . . . . .	200
7.3	Euler's Zeta Function . . . . .	210
7.4	Euler and the Fourier Series . . . . .	224
7.5	Generalized Zeta Function . . . . .	230
7.6	Applications of the Zeta Function to Mathematical Physics and Algebraic Geometry . . . . .	231
8.	Euler's Beta and Gamma Functions and Infinite Products	235
8.1	Introduction . . . . .	235
8.2	Euler's Beta and Gamma Functions . . . . .	236
8.3	Applications of the Euler Gamma Functions . . . . .	247
8.4	Euler's Contributions to Infinite Products . . . . .	248
9.	Euler and Differential Equations	255
9.1	Historical Introduction . . . . .	255
9.2	Euler's Contributions to Ordinary Differential Equations .	261
9.3	Euler's Work on Partial Differential Equations . . . . .	275
9.4	Euler and the Calculus of Variations . . . . .	288

10. The Euler Equations of Motion in Fluid Mechanics	297
10.1 Introduction . . . . .	297
10.2 Eulerian Descriptions of Fluid Flows . . . . .	298
11. Euler’s Contributions to Mechanics and Elasticity	309
11.1 Introduction . . . . .	309
11.2 Euler’s Work on Solid Mechanics . . . . .	312
11.3 Euler’s Research on Elastic Curves . . . . .	320
11.4 Impact of Euler’s Work on Modern Aerodynamics . . . . .	328
12. Euler’s Work on the Probability Theory	337
12.1 Introduction . . . . .	337
12.2 Euler’s Work on Probability . . . . .	341
12.3 Euler’s Beta and Gamma Density Distributions . . . . .	345
13. Euler’s Contributions to Ballistics	349
13.1 Introduction . . . . .	349
13.2 Euler’s Research on Ballistics . . . . .	352
14. Euler and his Work on Astronomy and Physics	359
14.1 Introduction . . . . .	359
14.2 Euler’s Contributions to Astronomy . . . . .	363
14.3 Euler’s Work on Physics . . . . .	369
<i>Bibliography</i>	373
<i>Index</i>	383