Preface

Leonhard Euler (1707-1783) was a universal genius and one of the most brilliant intellects of all time. He made numerous major contributions to eighteenth century pure and applied mathematics, solid and fluid mechanics, astronomy, physics, ballistics, celestial mechanics and optics. Among the greatest mathematical and physical scientists of all time including Newton, Leibniz, Gauss, Riemann, Hilbert, Poincaré, and Einstein, Euler's monumental contributions are generally considered unique and fundamental and have shaped much of the modern mathematical sciences. The Eulerian universal view is the dominant influence in the fields of physics, astronomy, continuum mechanics, natural philosophy, pure and applied mathematics. He published almost 900 original research papers, memoirs, and 25 books and treatises on mathematical and physical sciences. Even without the publication of his collected works, *Leonhardi Euleri Opera Omnia*, still in the process of being edited by the Swiss Academy of Sciences, his voluminous published works clearly demonstrate his amazing creativity, achievements and contributions to a wide variety of subjects in mathematical, physical, and engineering sciences. He also made contributions to other disciplines including geography, chemistry, cartography, music, history and philosophy of science.

The following quotations give some idea of the special veneration and affection in which he was held by his contemporaries and successors. P. S. Laplace wrote: "Read Euler, read Euler, he is the master of us all." It is a delight to quote Karl Friedrich Gauss: "... the study of Euler's works will remain the best school for different fields of mathematics and nothing else can replace it." On the other hand, the great twentieth century mathematician André Weil said: "No mathematician ever attained such a position of undisputed leadership in all branches of mathematics, pure and applied, as Euler did for the best part of the eighteenth century."

The tercentenary of Euler's birth has recently been celebrated with glorious success to pay a special tribute to this legendary mathematical and physical scientist of the eighteenth century. There is absolutely no doubt that Euler laid the solid foundations on which his contemporaries and successors of the last three centuries were able to build new ideas, results, theorems and proofs. His extraordinary genius created a simple language and style, unique symbols, and notations in which mathematical and physical sciences have developed ever since. His name is also synoymously associated with a large number of results, terms, equations, theorems, and proofs in mathematics and science.

Throughout his extensive research contributions and lucid writings, Euler was always influenced by his own thought as follows: "Since a general solution must be judged impossible from want of analysis, we must be content with the knowledge of some special cases, and that all the more, since the development of various cases seems to be the only way to bringing us at last to a more perfect knowledge." In addition, Euler's quest of new knowledge was simple and direct. His standards of mathematical rigor were far more primitive than those of today, but as Richard Feynman (1918-1988), an American genius, so cogently observed in the twentieth century: "... However, the emphasis should be somewhat more on how to do the mathematics quickly and easily, and what formulas are true, rather than the mathematician's interest in methods of rigorous proof." Euler has often been criticized for his lack of mathematical clarity, elegance and rigor. Intuition played an important role in his discoveries. He was always interested in creating a set of new ideas and results in the most diverse fields of mathematical and physical sciences. So, it is perhaps true that Euler's work met all requirements for rigor in his time. He was often satisfied when his intuition gave him full confidence that the proof of results could be carried through to complete mathematical rigor and then assigned the completion of the proof to others.

In pure mathematics, his major research fields included differential and integral calculus, infinite series and products, algebra, number theory, geometry of curves and surfaces, topology, graph theory, ordinary and partial differential equations, calculus of variations, special functions, elliptic functions, and integrals. In applied mathematics, he published papers on the mechanics of particles and of solid bodies, elasticity and fluid mechanics, optics, astronomy, lunar, and planetary motion. He also wrote many textbooks on mechanics, mathematical analysis, algebra, analytic geometry, differential geometry, and the calculus of variations. In mathematical

physics, Euler discovered the fundamental partial differential equations for the motion of inviscid incompressible and compressible fluid flows, and applied them to the blood flow in the human body. In the theory of heat, he closely followed Daniel Bernoulli to describe heat as an oscillation of molecules. He mathematically investigated the propagation of sound waves and obtained many original results on refraction and dispersion of light. Euler was one of the few scientists of the eighteenth century to favor the wave theory as opposed to the particle theory of light. Euler also made remarkable contributions to applied mathematics and engineering science. For example, he studied the bending of beams and calculated the critical load of columns. He described the perturbation effect of celestial bodies on the orbits of planets. He obtained the paths of projectiles in a resisting medium. He worked on the theory of tides and currents. His study on the design of ships helped navigation. His three volumes on achromatic optical instruments contributed to the design of microscopes and telescopes.

Euler maintained extensive contacts and correspondence with many of the most eminent mathematical scientists of the time including Christian Goldbach, A. C. Clairaut, Jean d'Alembert, Joseph Louis Lagrange, and Pierre Simon Laplace. This led to the development of personal and professional relationship between them. There was an amicable correspondence between Euler and Goldbach, and Euler and Clairaut which dealt with topical problems of number theory, mathematical analysis, differential equations, fluid mechanics, and celestial mechanics. There were neither any disagreements nor claims of one against the other. They discussed all mathematical ideas and problems openly, often significantly prior to their publication. Euler in Berlin and d'Alembert in Paris had an extensive mathematical correspondence over many years. In 1757, they had a strong disagreement, which eventually led to an estrangement, on whether discontinuous or non-differentiable functions are admissible solutions of the vibrating string problem. There was also a priority dispute between them on the theory of the precession of the equinoxes and nutation of the axis of the Earth. However, after d'Alembert visited Euler in Berlin in 1763, their relation became more cordial. In 1759, the young Lagrange joined in the discussion of solutions with a controversial article which was criticized by both Euler and d'Alembert. However, Lagrange sided with most of Euler's views. In 1761, Lagrange, seeking to meet the criticisms of d'Alembert and others, provided a different treatment of the vibrating string problem. The debate continued for another twenty years with no resolution. The issues in dispute were not resolved until Joseph Fourier picked up the subject in

the next century. Although Euler made an important and seminal contribution to calculus of variations, Lagrange, at the age of 19, made the first formulation of the equations of analytical dynamics according to the principles of the calculus of variations, and his approach was superior to Euler's semi-geometric methods. Thus, the classical Euler-Lagrange variational problem of determining the extremum value of a functional led to the celebrated Euler-Lagrange equation.

It has been calculated that his publications during his life averaged about 800 pages a year. His complete works entitled *Opera Omnia* consist of nearly 80 volumes, each approximately between 300 and 600 pages. Euler was undoubtedly the most prolific mathematical and physical scientists of all time. His whole working life was totally dedicated to the pursuit of fundamental discovery, dissemination of mathematical and scientific knowledge, and popularization of their value to common people. His famous three-volume *Letters to a German Princess on Different Subjects in Natural Philosophy* was one of the most popular books on science ever written and it was translated from German into eight different languages. The *Letters* addressed a wide variety of subjects including optics, acoustics, mechanics, astronomy, music, dioptrics, electricity and magnetism. This publication was essentially a unique encyclopedia of physical and philosophical ideas written in a popular style for the widest possible common audience. This work formed the basis for the reform of the teaching of physics and science. These are just a few examples of his prodigious contributions.

This volume is intended as a tricentennial memorial tribute to this universal mathematical scientist. My desire as well as interest in writing this book commemorating Euler's major contributions to mathematical and physical sciences is founded on the deep respect and admiration for him that I have gained from my own study and research of a small fragment of his voluminous work. The origin of this book was essentially based on my postgraduate course in the theory of elliptic functions and integrals with applications in 1960s. Indeed, I was further stimulated by my own articles and lectures for the last ten years on Euler and his major contributions. These publications and presentations are intended for the great majority of senior undergraduates and graduate students of mathematics, physics, and engineering.

The intense and narrow specialization of contemporary mathematics is a fairly recent phenomenon. The professional mathematical scientists spend almost all of their time and energy on segments of mathematics or science that seem to have little relationship to each other. They have hardly any

time or opportunity to become familiar with the history of mathematics and science. The emphasis on history may provide more broad perspective on the whole subject and relate the subject matter of the courses not only to each other, but also to the major developments of mathematical thoughts. As Henri Poincaré eloquently wrote: "If you wish to foresee the future of mathematics, our proper course is to study the history and present condition of science." The writing of this volume was greatly influenced by the above thought of Poincaré. This book may serve to some extent as a historical introduction to mathematical sciences with the major emphasis on selected Euler's contributions. I hope that it will be helpful to professional and prospective mathematical scientists.

While writing this book as an exposition and survey of history, three major objectives have been kept in mind. The first is to focus each chapter on a subject to which Euler made a significant research contribution. Included are a short history of mathematical developments and discoveries before Euler, and a brief sketch of the life, work, career, and major achievements of Euler. The second is to present some historically significant, elegant, or unexpected theorems, proofs and results with applications. The third is to convey something of the fascination of mathematical sciences — of their beauty, intellectual power, and wide variety. This book does not require a graduate school mastery of any branch of mathematical sciences. It contains a wide variety of material accessible to the widest possible audience of mathematically literate readers.

It is my pleasure to express my grateful thanks to many friends, professional colleagues and students around the world who offered their suggestions and help at various stages of the preparation of the book. I am particularly grateful to my graduate students, Arunabha Biswas and Arindam Roy for helping me during the preparation of the book, especially for drawing all the figures in the book. My special thanks to Ms. Veronica Chavarria who cheerfully typed the manuscript with constant changes and revisions and carefully checked all the names in the text. In spite of the best efforts of everyone involved, some typographical errors will doubtlessly remain. I wish to express thanks to Ms. Lai Fun Kwong and the Production Department of Imperial College Press for their help and cooperation. Finally, I am deeply indebted to my wife, Sadhana, for her understanding and tolerance while the book was being written.

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