Preface

> 'Uncertainties appear everywhere in the model. When using a mathematical model, careful attention must be given to the uncertainties in the model.' (Feynman, 1988)
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> 'New methods for treating uncertainty will become important in virtually all branches of mechanics.' (Oden, Belytschko, Babuška and Hughes, 2003)
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> 'Scientific progress usually can only be achieved by experts of different disciplines working together.' (Eschenauer and Olhoff, 2001)
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> 'The ethics of our profession today does not allow any design for a structure without optimization.' (Mungan, 2001)

Researchers in applied mechanics, whether in the field of aerospace, architecture, civil, mechanical or ocean engineering, invariably adopt the *either/or* style. Namely, they devote themselves either to linear or to nonlinear analysis of the structure they are dealing with, they are engaged in analyzing it either in the elastic or in the inelastic range; they deal either with its static or with its dynamic behavior.

Along the same lines, researchers classify themselves according to the deterministic or non-deterministic nature of the problems they are tackling. The former stipulate that the geometry, loads, boundary conditions, material characteristics involved in a problem are fully specified; they stress that they are concerned with understanding the phenomenon at hand, and then providing methods of solution. To achieve such a basic comprehension of what is going on, they make (often far-reaching) assumptions: that the parameters involved are fully specified, that they take on large or small values, etc. Such an approach, which may seem unjustifiable at first glance, often leads to breakthrough discoveries in understanding the phenomenon under study.

The difference between the deterministic and non-deterministic approaches lies in the fact that the deterministic design attempts to retain its determinism until the very last stage of design, whereas the non-deterministic analysts maintain that there are no two structures with identical material and geometric properties, boundary conditions, or loading and, therefore, uncertainty analysis is called for.

In reality, this *deterministic versus non-deterministic* attitude is a fallacy. To justify this *undiplomatic* statement, we need deeper insight into the essence of deterministic analysis. In it, after careful evaluation of displacements, strains, and stresses (often with many significant digits – since computer outputs consist of numbers), the engineers compare the latter against the allowable stress; i.e., the experimentally measured yield- or ultimate-level multiplied by some *fudge* coefficient called *the safety factor* (as in 'better safe than sorry'). Use of this factor in textbooks on engineering mechanics is invariably motivated by insufficient knowledge (don't we always have to make analytic assumptions of various sorts?), or by uncertainty (yes, deterministic analysts do pay lip service to the property they claim to neglect!) in loads, geometric characteristics, or scatter in material properties.

Thus, deterministic analysis, strictly speaking, is not as deterministic as advertised. Within it, uncertainty is introduced as *dessert* or, as in Bolotin's words (Bolotin, 1961), *via the back door*.

By contrast, in non-deterministic design, uncertainty figures as a legitimate ingredient throughout the whole design process, specifically during the thinking, analysis and design stages.

One may conclude, therefore, that non-deterministic analysis is more *honest* than its so-called deterministic counterpart as it *places its cards on the table* from the very beginning. As for the latter, in view of the above reasoning, it would be appropriate to dub it *pseudo-deterministic* rather than deterministic. If experience enables us to establish the precise value of a safety factor, then the pseudo-deterministic unknowns are nothing less than the lumped, integral equivalent of the inherently present uncertainty in engineering problems.

Naturally, the advent of the safety factor was at the time a major breakthrough in design. Despite its obvious arbitrariness, it allowed us to put up a wall against failure. At the same time, this dogmatism implies that if the actual stress turns out to be below the allowable level, then there can be no failure, and the structure will be safe throughout its exploitation.

But how can we establish the safety factor, except by experience or trial-and-error (one could sarcastically ask at this stage: is this a trial or an error?). According to Norton (2000), 'choosing the safety factor is often a confusing proposition for the beginning engineer,' and not only for the beginning one, we may add. According to Bruhn (1975),

> '[the safety] factor is designed to arbitrarily account for items such as material properties, manufacturing differences, since no two parts can be made exactly the same; uncertainties in the loading environment; and unknowns in the internal load and stress distributions.'

As we observe, even with the manner of explanation of the safety factor, this concept is quite critical within the semi-deterministic framework for it utilizes adjectives like *arbitrary* (Bruhn, 1975) and *confusing* (Norton, 2000).

Freudenthal (1968) emphasized:

> 'It seems absurd to strive for more and more refinement of methods of stress analysis, if in order to determine the dimension of the structural elements, its results are subsequently compared with a so-called working stress, derived in a rather crude manner by dividing the values of somewhat dubious material parameters obtained in a conventional material test by a still more dubious empirical number, called a safety factor.'

Bolotin (1961) had written,

> 'The values of safety factors, as well as closely associated values of design loads and design resistances, were imposed and modified mainly empirically, by way of generalization of long-term experience in exploitation of structures. Yet, as is seen from the essence of the problem, there are in principle, also theoretical approaches possible, with wide application of the theory of probability and mathematical statistics.'

We must immediately note that probability theory and mathematical statistics are not the only avenues for the theoretical approach to the safety factors.

Non-deterministic analysis enables us to recast the safety factor approach in terms of other quantities. For example, Freudenthal (1957) and Rzhanitsyn (1947) pioneered an interpretation of the safety factor in terms of reliability based on the probability theory and mathematical statistics, and introduced the so-called *central safety factor*. (It was shown by Elishakoff (2004) that probabilistic mechanics offers four possible interpretations of the safety-factor concept.)

Probabilistic analysis is not the only game in town, in the non-deterministic context. While the probability densities of the random variables involved in it are often not known, simpler characteristics like the sets within which the parameters vary – may be specified. Such a description of uncertainty (Bulgakov, 1942) is referred to as an unknown-but-bounded one. In the West it was independently developed by Boley (1966a) for static thermoelastic problems; and by Drenick (1968) and Shinozuka (1970) for dynamic ones. Some of these and other results in applied mechanics were summarized by Ben-Haim and Elishakoff (1990), who also developed some new material. They concentrated their attention on models of uncertainty in which the variations were treated as convex sets, and yielded the most and least favorable stresses, strains or displacements over such a convex set. This approach has been extended by Elishakoff, Lin and Zhu (1994e) by combining it with probabilistic analysis; and by Elishakoff, Li and Starnes (2001) via systematic comparison with the probabilistic methodology.

Elishakoff (1994) interpreted the safety factor within unknown-but-bounded uncertainty as the interval $[n_{\mathrm{L}}, n_{\mathrm{U}}]$, between lower 'L' and upper 'U' bounds representing the ratios of the yield stress to the least and most favorable response, respectively, in which case the yield stress can safely serve as the design parameter. Likewise, Elishakoff and Ferracuti (2006a, 2006b) provided interpretation of the safety factor within fuzzy sets. Here, too, as in probabilistic analysis, there are four possible interpretations of the fuzzy safety factor.

This book is exclusively concerned with the unknown-but-bounded uncertainty. Chapter 1 explains in detail why this approach was chosen. Here, we content ourselves with noting that in the case of random variables with bounded support, it is shown that probabilistic design and the unknown-but-bounded approach yield coincident results, when the required reliability tends to unity. Accordingly the second version was chosen as the less complicated of the two, in keeping with Albert Einstein's sage precept: 'Everything should be made as simple as possible, but not simpler.' In the past decade or so, many papers were devoted to unknown-but-bounded uncertainty in applied mechanics. Several books were published as well. Regrettably, some of the authors, instead of concentrating on the safety factor as the main tool of structural design and professional communication, turned to alternative concepts or actually introduced new (and hardly needed) ones, thereby diverting research activities from the really relevant problems (see the instructive title of the book by Sokal and Bricmont (1998)).

We would like next to consider the *optimization/anti-optimization problem*, but before proceeding further it would be instructive to reproduce the text of a letter sent by Dr. Rudolf F. Drenick (2001), Professor Emeritus of Brooklyn Polytechnic Institute, to Dr. Izuru Takewaki, Professor of Kyoto University:

> 'You might be amused by the story of how I came to do the work on the critical excitation. In (I think) 1965 I participated in a Japan/U.S. workshop on applied stochastics. After two days of lectures, one of the observers at our meeting made some comments which I found very interesting. He said something like this. "My name is Ozawa and I work for the Tokyo Building Licensing Bureau. I am forever visited by architects who show me plans of buildings they plan to construct and I am supposed to tell them whether their plans are good or bad, and I have never really known what to tell them. Now I have listened to you gentlemen for two days and I still don't know what to tell them." There followed a great deal of discussion among the workshop participants but in the end the discussion leader, Professor Bogdanov, said "I am very sorry, Mr. Ozawa, but we can't help you."
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> I went home from the workshop and began to think of how I could

> deal with Mr. Ozawa's problem. I concluded that any method I should suggest to him would have to be, first of all, practical. I also felt that it should be as distribution-free as possible. After some thought the concept of the critical excitation came to me. (The term *critical excitation* was not my idea. It was suggested by Prof. Penzien.) My first results were largely distribution-free, as you know, but they were not practical. They were much too conservative to be useful.'

This worst-case design was never adopted by the nuclear industry because of its extreme conservatism, as Drenick informed one of us (I.E.). He, accordingly, became very pessimistic about the likelihood of the engineering profession ever adopting it. He was however very encouraging, and so kind as to write the foreword to the book by Ben-Haim and Elishakoff (1990) in which he stated perhaps with some measure of exaggeration: 'Their approach is novel and highly welcome. In my opinion, it is inevitable that it, and its extensions, will dominate the future practice of engineering.'

In consequence of the above-mentioned book, it was realized by one of us (I.E.) that in order to make this worst-case scenario research practical, it needed an infusion of new blood. (It was realized that unknown-but-bounded uncertainty analysis is a *defacto* as *anti-optimization* process, the reverse of searching for the best solution.) It became also apparent that it is impractical to content oneself with the least favorable static or dynamic response, or with the worst possible buckling load. Rather a structure should be so designed as to minimize the first and maximize the second – in other words, adherence to the time-honored precept of 'making the best out of the worst.' It is gratifying to read that this sentiment is now shared by other investigators, for example, Ben-Tal and Nemirovski (2002):

> 'The *worst-case oriented* interval model of uncertainty looks *too conservative*. Indeed, when the perturbations in coefficients of an uncertain linear inequality are of stochastic nature, it is *highly improbable* that they will *simultaneously* take the *most dangerous* values.'

These considerations eventually gave rise to the *optimization and anti-optimization* terminology or better yet *optimization with anti-optimization*. If there is an uncertainty, one should, as it were, participate in two dances concurrently, doing both *optimization* and *anti-optimization* steps.

Such a hybrid approach would be less conservative than its worst-scenario counterpart, hence is much more likely to gain credibility and acceptance among researchers and practicing engineers.

Indeed, it is often said that if a researcher cannot explain his or her research to a lay person, then that researcher does not yet fully understand the subject under discussion (Einstein, here too, comes handy: 'you do not really understand something unless you can explain it to your grandmother').

Indeed, would not it be an engineering *sin*, as it were, not to improve the worst scenario and leave it intact – especially if one has to save expenditure? Only very rich companies, or those in positions of exceptional responsibility, can afford exclusive reliance on anti-optimization.

One argument against optimization is that if it is applied to a specific loading condition, the result may prove unsatisfactory under a different condition. In such a case, it suffices to add the latter and repeat the procedure for the new circumstances. Alternatively, one can introduce uncertainty and assign bounds so as to obtain the optimal solution under the worst-case scenario.

Another criticism concerns optimization against nonlinear buckling. As explained in detail in Chap. 5, optimization against buckling does not always yield imperfection-sensitive structures, the resulting sensitivity being in fact lower than that of a non-optimal design (Ohsaki, 2002c). However, the buckling load of a perfect system is drastically increased by optimization, and its counterpart for an imperfect system is little affected by sensitivity changes.

The combined optimistic-pessimistic approach can be characterized as follows: optimists build ships; pessimists build lifeboats. Both are needed for safe sailing. Recall the overly optimistic view on the *Titanic*, which was claimed to be so robust that even God couldn't sink it! As a result, an insufficient number of lifeboats was on board during its maiden voyage. This complacency cost over 1,500 lives.

We advocate a combination of healthy doses of optimism (*represented* by optimization) and pessimism (anti-optimization).

The above discussion gave rise to the notion that uncertainty analysis is reducible to one of the three vertices of the *uncertainty triangle* (Elishakoff, 1990, 1998b), shown below. These three approaches do not represent three non-interesting magistrata!

Although the first paper co-authored by one of us (Ben-Haim and Elishakoff, 1989b) was presented at two conferences in 1988 and appeared in 1989, we had been able, on an earlier occasion (Elishakoff, 1983, p. 42), to compare the probabilistic and worst-case designs on a simple example included on the 'Statistical Methods in Elasticity' course at the Technion - Israel Institute of Technology since 1973. The monograph by Ben-Haim and Elishakoff (1990) did not correlate probabilistic and convex modelings. Only later, in the paper by Elishakoff, Cai and Starnes (1994a), was a direct comparison made between probabilistic and anti-optimization techniques on the nonlinear boundary-value problem (see also its generalization by Qiu, Ma and Wang (2006a)). It turned out that if the probabilistic information is available and the required reliability is not excessively high, anti-optimization may be conservative. However, if near-unity reliability is required, the two approaches tend to yield close or coincident results.

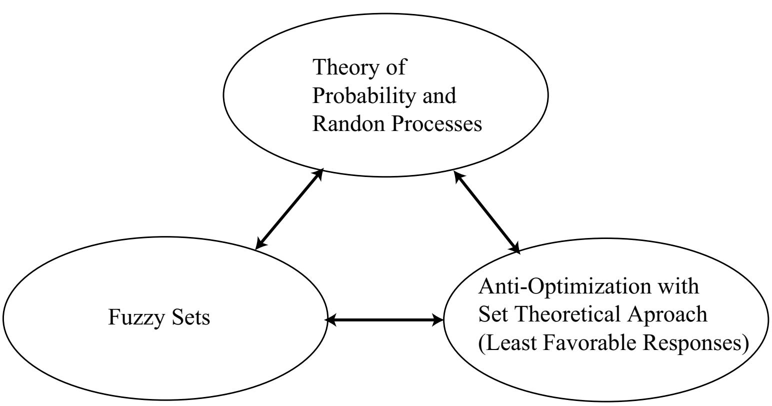
It was also realized that these seemingly competing approaches are nevertheless amenable to a certain measure of *cooperation*. An instructive example is the case of the turbulent excitation experienced by the space shuttle weather protection systems at the Kennedy Space Center. This excitation is inherently random and must be analyzed probabilistically, but inadequate data is available on its cross-spectral density. This shortage was remedied (Elishakoff, Lin and Zhu, 1994e) by means of the anti-optimization approach, which yielded the least favorable mean-square displacement of the weather protection panels.

In another combination, Kosko (1990), Maglaras, Nikolaidis, Haftka and Gudney (1997), Chiang, Dong and Wong (1987), and Wood, Antonnson and Beck (1990) watched the fuzzy-sets approach against the probabilistic one, theoretically and/or experimentally. Kwakernaak (1978), Haldar and Reddy (1992), Savoia (2002), and Möller and Beer (2004) combined the two approaches. Finally, Fang, Smith and Elishakoff (1998) and Tonon, Bernardini and Elishakoff (2001) utilized *all* three approaches for their analyses.

Bernardini (1999) and Tonon, Bae, Grandhi and Pettit (2006) claim that all three methodologies stem from a single theory of random sets (see Matheron (1975) and Dubois and Prade (1991)). Recently, Ben-Haim (2001) also adopted combined probabilistic and non-probabilistic analyses.

As is clearly seen from the above, probabilistic and non-probabilistic analyses of uncertainty are not *irreconcilable* and can complement each other when deemed useful. Such *reconciliation* was in fact advocated by Elishakoff (1995b):

> 'Convex modeling of uncertainty, and in general the set-theoretic modeling of it (the constraints should not necessarily be treated as convex), do not support probabilistic ideas. Convex modeling rather complements both the probabilistic approach and the fuzzy subsets based treatment.'



Uncertainty triangle (Elishakoff, 1990).

One should constantly bear in mind that just as pseudo-deterministic analysis incorporates uncertainty via a lumped number (namely the safety factor), the probabilistic approach is partly deterministic, as not all parameters are considered variable. Does one have to consider all parameters of a structure as uncertain, thereby covering the general case? Some research establishments take this line and present on their slides perhaps tens of random variables with arbitrary probabilistic distributions to choose from! Its Excellency the Computer then provides the desired output in seconds. Such excessive zeal is unjustifiable, seeing that no probabilistic dependencies are either known or incorporated into the programs! As Babuška, Nobile and Tempone (2005) emphasize,

> 'whenever the input data belong to an infinite dimensional space (they might be functions of position and/or time), their probabilistic characterization must include knowledge of the cross-correlation of the values that the data can take at different points in space and time. In this case, the solution of a stochastic model becomes quickly too costly.'

Still, we should be thankful that the innocent number π, for example, is not declared a random variable with mean value 3.14... and zero variance! One should refrain from considering all of them as random variables unless they have sufficiently large variability to justify the non-deterministic approach. Likewise, one should abandon the idea (attractive for beginners) of treating all parameters as uncertain, even if some of them have sufficiently small variability.

In conclusion, one has to maintain a delicate and healthy balance in modeling a system. Bolotin (1969), for example, is against both underestimation and overestimation of uncertainty analysis. We hope that this book will serve as a beacon for practicing engineers and researchers, and help them reevaluate their thinking and practices on uncertainty analysis.

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