

# Contents

Preface	vii
1. Introduction	1
1.1 The Logical Space of Meaning . . . . .	1
1.2 The Aim of This Book . . . . .	4
1.3 Starting from Traditional Formal Semantics . . . . .	4
1.4 Semantics and the History of Logic (1): Intuitionism . .	5
1.4.1 Curry–Howard . . . . .	5
1.4.2 Lambek and the substructural hypothesis . . .	5
1.5 Semantics and the History of Logic (2): Classicism . . .	6
1.6 Semantics and the History of Logic (3): Linear Logic . .	7
1.7 Presentation of the Book . . . . .	10

## Part I. Truth-Conditional Meaning

2. Compositional Approaches and Binding	15
2.1 Representing Logical Meaning: The Binding Issue . . .	15
2.1.1 The syntactic notion of binding . . . . .	15
2.1.2 The semantic notion of binding . . . . .	18
2.1.3 The model-theoretic notion of binding . . . . .	18
2.2 Syntactic Derivations and Semantic Composition . . . .	20
2.3 Montague Grammar Revisited . . . . .	20
2.3.1 From rules to sequents . . . . .	20
2.3.2 On relatives and quantification . . . . .	24

2.3.3	Examples . . . . .	25
2.3.4	On binding . . . . .	29
2.4	A Theory of Simple Types . . . . .	30
2.5	Heim and Kratzer's Theory . . . . .	33
2.5.1	Interpreting derivation trees . . . . .	33
2.5.2	Predicate modification . . . . .	38
2.5.3	Variables and binding . . . . .	39
2.5.4	Towards a proof-theoretic account of binding . . . . .	50
3.	Derivationalism . . . . .	53
3.1	Introduction . . . . .	53
3.2	Categorial Grammars . . . . .	54
3.3	The (Pure) Lambek Calculus . . . . .	60
3.3.1	The mathematics of sentence structure . . . . .	60
3.3.2	A categorial system . . . . .	61
3.4	Minimalist Grammars . . . . .	61
3.4.1	Minimalist principles . . . . .	61
3.4.2	Features . . . . .	65
3.4.3	Minimalist grammars . . . . .	66
3.4.4	Merge . . . . .	66
3.4.5	Move . . . . .	67
3.4.6	Minimalist grammars and categorial grammars . . . . .	73
3.4.7	Binding as "Cooper storage" . . . . .	74
3.4.8	The interpretation of derivational trees . . . . .	75
3.4.9	The semantic interpretation of Merge . . . . .	76
3.4.10	The semantic interpretation of Move . . . . .	78
3.5	Concluding Remarks . . . . .	80

## Part II. Logic

4.	Deductive Systems . . . . .	85
4.1	Fitch's Natural Deduction System . . . . .	85
4.1.1	Conjunction . . . . .	87
4.1.2	Implication . . . . .	88

4.1.3	Disjunction . . . . .	89
4.1.4	Negation . . . . .	90
4.2	Natural Deduction in Intuitionistic Logic . . . . .	91
4.2.1	Tree format . . . . .	91
4.2.2	Normalization . . . . .	92
4.2.3	Sequent format . . . . .	93
4.3	Intuitionistic Sequent Calculus . . . . .	94
4.3.1	Structural rule . . . . .	94
4.3.2	Identity rules . . . . .	95
4.3.3	Logical rules . . . . .	95
4.3.4	An example of a proof in intuitionistic logic . . . . .	96
4.3.5	The cut rule . . . . .	97
4.3.6	Lists and sets . . . . .	98
4.3.7	Structural rules . . . . .	98
4.4	Classical Sequent Calculus . . . . .	99
4.4.1	Structural rules . . . . .	99
4.4.2	Identity rules . . . . .	100
4.4.3	Logical rules . . . . .	100
4.5	Some Properties of the Sequent Calculus . . . . .	102
4.5.1	Subformula property . . . . .	102
4.5.2	Cut-elimination . . . . .	103
4.6	Linear Logic . . . . .	103
4.6.1	Identity rules . . . . .	103
4.6.2	Logical rules . . . . .	104
4.6.3	Exponentials . . . . .	105
4.6.4	Constants . . . . .	107
4.6.5	The one-sided calculus . . . . .	108
4.6.6	Intuitive interpretation . . . . .	109
4.7	Back to the Lambek Calculus . . . . .	115
4.7.1	The Lambek calculus as non-commutative linear logic . . . . .	115
4.8	Linguistic Applications of the Additives . . . . .	118
4.9	Proof Nets . . . . .	119
4.9.1	A geometrization of logic . . . . .	119
4.9.2	Cut-elimination in proof nets . . . . .	124
4.10	Proof Nets for the Lambek Calculus . . . . .	125
4.11	Concluding Remarks . . . . .	126

5.	Curry–Howard Correspondence	127
5.1	Introduction . . . . .	127
5.2	A Correspondence Between Types and Formula . . . . .	128
5.3	An Example of a Combinator . . . . .	132
5.4	Concluding Remarks . . . . .	134

### Part III. Proof Theory Applied to Linguistics

6.	Using the Lambek Calculus and Its Variants	137
6.1	Introduction . . . . .	137
6.2	Using the Lambek Calculus . . . . .	137
6.2.1	Summary of the rules . . . . .	137
6.2.2	Natural deduction presentation . . . . .	138
6.2.3	Examples . . . . .	139
6.2.4	Compositional semantics . . . . .	143
6.2.5	Limitations of the Lambek calculus . . . . .	145
6.3	Flexible Types . . . . .	148
6.3.1	Flexible Montague grammar . . . . .	148
6.3.2	Variable-free semantics . . . . .	149
6.4	Non-associative Lambek Calculus . . . . .	151
6.5	Semantics of the Lambek Calculus . . . . .	153
6.5.1	Monoidal semantics . . . . .	153
6.5.2	Relational semantics . . . . .	154
6.6	An Extension of the Lambek Calculus:	
	The Lambek–Grishin Calculus . . . . .	157
6.6.1	Adding new connectives . . . . .	157
6.6.2	Rules . . . . .	158
6.6.3	The Grishin postulates . . . . .	159
7.	Grammatical Reasoning	163
7.1	Motivations . . . . .	163
7.2	Modal Preliminary . . . . .	164
7.2.1	Necessity and possibility . . . . .	164
7.2.2	Axiomatization . . . . .	164
7.3	Residuation and Modalities . . . . .	166
7.4	Linguistic Applications . . . . .	169
7.5	Back to Quantification . . . . .	171

7.6	Kripke Semantics . . . . .	173
7.7	Concluding Remarks and Observations . . . . .	174
8.	A Type-Theoretical Version of Minimalist Grammars . . . . .	177
8.1	Inserting Chains . . . . .	177
8.2	Head Movement . . . . .	189
8.3	Adjoining and Scrambling . . . . .	190
8.4	Semantics Without Cooper Storage . . . . .	193
	8.4.1 Cooper storage and hypothetical reasoning . . . . .	193
	8.4.2 One or two derivations? . . . . .	195
8.5	Concluding Remarks: Some Tracks to Explore . . . . .	201
9.	Grammars in Deductive Forms . . . . .	203
9.1	Introduction . . . . .	203
9.2	Convergent Grammars . . . . .	203
	9.2.1 CVG types and rules: Semantics . . . . .	203
	9.2.2 CVG types and rules: Syntax . . . . .	205
	9.2.3 An example . . . . .	206
9.3	Labelled Linear Grammars . . . . .	206
9.4	Binding in LLG . . . . .	215
	9.4.1 Pronouns as pronounced variables . . . . .	215
	9.4.2 On reflexives . . . . .	220
9.5	On Phases . . . . .	220
9.6	Comparing CVG and LLG . . . . .	221
9.7	Concluding Remarks . . . . .	222
10.	Continuations and Contexts . . . . .	223
10.1	The Use of Continuations in Semantics . . . . .	223
	10.1.1 Continuations and quantification . . . . .	223
	10.1.2 Continuizing a grammar . . . . .	224
	10.1.3 Call-by-value and call-by-name . . . . .	228
10.2	Symmetric Calculi . . . . .	232
	10.2.1 Towards the $\lambda\mu$ -calculus . . . . .	232
	10.2.2 Applications of the $\lambda\mu$ -calculus to the problem of scope construal . . . . .	237
	10.2.3 $\lambda\mu\tilde{\mu}$ -calculus . . . . .	239
	10.2.4 Call-by-name vs call-by-value . . . . .	243
	10.2.5 Back to implication . . . . .	245

10.2.6	Substraction . . . . .	246
10.2.7	Back to links between logic and duality . . . . .	247
10.2.8	From the $\lambda\mu$ -calculus to the $\lambda\mu\tilde{\mu}$ -calculus . . . . .	256
10.2.9	A small linguistic example . . . . .	256
10.3	Concluding Remarks and Further Works . . . . .	259
11.	Proofs as Meanings . . . . .	263
11.1	From Intuitionistic Logic to Constructive Type Theory . . . . .	263
11.1.1	Proof processes and proof objects . . . . .	263
11.1.2	Judgements . . . . .	266
11.1.3	Dependent types . . . . .	271
11.1.4	Contexts . . . . .	274
11.2	Formalizing Montague Grammar in Constructive Type Theory . . . . .	278
11.2.1	Predicate calculus . . . . .	279
11.2.2	Translation into predicate calculus . . . . .	280
11.2.3	Introducing dependent types . . . . .	281
11.3	Dynamical Interpretation and Anaphoric Expressions . . . . .	282
11.3.1	<i>If ... then</i> sentences . . . . .	282
11.3.2	A presuppositional account . . . . .	285
11.4	From Sentences to Dialogue . . . . .	287

## Part IV. Ludics

12.	Interaction and Dialogue . . . . .	291
12.1	Dialogue and Games . . . . .	291
12.1.1	Introduction . . . . .	291
12.1.2	Game semantics . . . . .	292
12.1.3	Internal games . . . . .	300
12.1.4	Comparison between the two frameworks . . . . .	304
12.2	Ludics . . . . .	306
12.2.1	Focusing proofs . . . . .	306
12.2.2	Paraproofs . . . . .	310
12.2.3	An easy application to natural language . . . . .	314
12.2.4	Designs . . . . .	317

12.3	Behaviours . . . . .	328
12.3.1	Some particular behaviours . . . . .	328
12.3.2	What logic for ludics? . . . . .	330
12.3.3	Sums and products . . . . .	332
12.3.4	Discursive relations . . . . .	337
13.	The Future in Conclusion	347
	Bibliography	353
	General Index	363
	Author Index	367