Preface

Mathematical finance theory has been established as one sector of finance theory. This theory is based on probability theory and many probabilists have contributed to this field.

The Black–Scholes model is a typical model for the complete market. This model is an outstanding model convenient to analyze, but it is well-known that in the real world the completeness of the market is not usually satisfied. The distribution of the log return of an asset usually has a fat tail and asymmetry, and the market is usually incomplete. Therefore we need other models for the incomplete market.

The *geometric Lévy process* (GLP) model is one of the most important models for the incomplete market. This model is able to possess a fat tail property, an asymmetric distribution and smile/smirk properties of implied volatility.

An incomplete market has many martingale measures (or risk-neutral measures) by the second fundamental theorem of mathematical finance. So we have to select a suitable martingale measure among them in order to discuss option pricing based on arbitrage theory. Many kinds of martingale measures have been proposed for this. Among them the minimal entropy martingale measure (MEMM) is the most important candidate.

Among the many GLP models, the importance of the *geometric stable process* (GSP) model is recognized by Fama [36] and Mandelbrot [77]. Since then many researchers have studied this model. The suitable martingale measure for the GSP model had not been evident for many years, but since the existence of the MEMM for GSP model was proved and an explicit form of it was obtained, the GSP model can be applied to a wide class of problems concerning option pricing.

The main subject of this book is the description of the [GLP & MEMM] (geometric Lévy process and minimal entropy martingale measure) option pricing models. We introduce these models and explain how to apply them to practical problems. This will be covered in Chapters 7, 8, and 9.

Before we introduce the [GLP & MEMM] option pricing models, we will explain the underlying ideas needed to understand our models. In Chapter 1 we give basic concepts in mathematical finance theory. In Chapter 2 Lévy processes and GLP models are introduced and described briefly. The following four chapters (from Chapter 3 to Chapter 6) are devoted to the investigation of martingale measures. In those chapters we stress the importance of Esscher-transformed martingale measures and minimal distance martingale measures. We also show that the MEMM is a special martingale measure among the set of all martingale measures in the sense that it is the minimal distance martingale measure determined by the entropy distance, and it is also an Esscher-transformed martingale measure determined by the simple risk process.

The importance of the [GLP & MEMM] option pricing models will be explained in Chapters 7 and 8. Furthermore it is explained in Chapter 9 that the [GSP & MEMM] model should be the most important one.

In this book we mainly treat a one-dimensional case. But many of the results for one-dimensional cases can be extended to multi-dimensional cases. In Chapter 10 we study a multi-dimensional case. The results obtained there are not simple extensions of the one-dimensional case and contain new concepts, new problems, and new ideas which are inherent to the multi-dimensional case. These days, a theory for non-arbitrage-free markets is needed. The idea of risk measure or value measure is an attempt to construct this theory. We introduce the idea of a risk-sensitive value measure and apply it to the problems of portfolio evaluation.

Appendix A is devoted to the explanation of the generalized method of moments for the parameter estimation of Lévy processes. The estimation problem of Lévy processes is related to calibration problems.

I hope that this book will be helpful to researchers, students, practitioners, and engineers of mathematical finance or risk management. This book is written for academics with a solid knowledge in "classical financial mathematics" and also for advanced practitioners who seek an introduction to the topic of option pricing in incomplete markets and into the recent Lévy process modeling. The necessary preliminary knowledge to read the book is briefly stated in Chapters 1 and 2. If a reader has a difficulty in following these chapters, I recommend first studying some of the books referred to in the Notes of Chapters 1 and 2.

I have been studying the [GLP & MEMM] models for more than a decade. During this period I have collaborated with Alexander Novikov, Tsukasa Fujiwara, Monique Jeanblanc, Susanne Klöppel, Tetsuya Misawa, Yoshiki Tsujii, and Naruhiko Moriwaki. Almost all of the results that were obtained through joint works with these people are contained in this book. Without these joint works this book could not have been published. I thank these colleagues from the bottom of my heart.

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