

Prefaces to the English Edition

Preface to the second edition

There are only few changes with respect to the first edition. The main ones are the following.

- 1) Chapter 4 has been expanded.
- 2) I added 27 new exercises with their solutions, making a total of 103 exercises.

While preparing this second edition, I benefited of several comments from S. Bandyopadhyay, G. Cupini, O. Kneuss and K.D. Semmler. I also want to thank the staff of Imperial College Press and World Scientific for their very nice job.

Preface to the first edition

The present monograph is a translation of *Introduction au calcul des variations* that was published by Presses Polytechniques et Universitaires Romandes. In fact it is more than a translation, it can be considered as a new edition. Indeed, I have substantially modified many proofs and exercises, with their solutions, adding also several new ones. In doing so I have benefited from many comments of students and colleagues who used the French version in their courses on the calculus of variations.

After several years of experience, I think that the present book can adequately serve as a concise and broad introduction to the calculus of variations. It can be used at undergraduate as well as at graduate level. Of course at a more advanced level it has to be complemented by more specialized materials and I have indicated, in every chapter, appropriate books for further readings. The numerous exercises, integrally solved in Chapter 7, will also be important to help understanding the subject better.

I would like to thank all students and colleagues for their comments on the French version, in particular O. Besson and M. M. Marques who commented in writing. Ms. M. F. DeCarminé helped me by efficiently typing the manuscript. Finally my thanks go to C. Hebeisen for the drawing of the figures.

Preface to the French Edition

The present book is a result of a graduate course that I taught at the Ecole Polytechnique Fédérale of Lausanne during the winter semester of 1990 - 1991.

The calculus of variations is one of the classical subjects in mathematics. Several outstanding mathematicians have contributed, over several centuries, to its development. It is still a very alive and evolving subject. Besides its mathematical importance and its links with other branches of mathematics, such as geometry or differential equations, it is widely used in physics, engineering, economics and biology. I have decided, in order to remain as unified and concise as possible, not to speak of any applications other than mathematical ones. Every interested reader, whether physicist, engineer or biologist, will easily see where, in his own subject, the results of the present monograph are used. This fact is clearly asserted by the numerous engineers and physicists that followed the course that resulted in the present book.

Let us now examine the content of the monograph. It should first be emphasized that it is not a reference book. Every individual chapter can be, on its own, the subject of a book. For example, I have written one that, essentially, covers the subject of Chapter 3. Furthermore several aspects of the calculus of variations are not discussed here. One of the aims is to serve as a guide in the extensive existing literature. However, the main purpose is to help the non-specialist, whether mathematician, physicist, engineer, student or researcher, to discover the most important problems, results and techniques of the subject. Despite the aim of addressing the non-specialists, I have tried not to sacrifice the mathematical rigor. Most of the theorems are either fully proved or proved under stronger, but significant, assumptions than stated.

The different chapters may be read more or less independently. In Chapter 1, I have recalled some standard results on spaces of functions (continuous, L^p or Sobolev spaces) and on convex analysis. The reader, familiar or not with these subjects, can, at first reading, omit this chapter and refer to it when needed in

the next ones. It is much used in Chapters 3 and 4 but less in the others. All of them, besides numerous examples, contain exercises that are fully solved in Chapter 7.

Finally I would like to thank the students and assistants that followed my course; their interest has been a strong motivation for writing these notes. I would like to thank J. Sesiano for several discussions concerning the history of the calculus of variations, F. Weissbaum for the figures contained in the book and S. D. Chatterji who accepted my manuscript in his collection at Presses Polytechniques et Universitaires Romandes (PPUR). My thanks also go to the staff of PPUR for their excellent job.