CHAPTER 1

ENERGY AND POWER

Many of the calculations involve conversions between different systems of units. The conversion factors are quoted from Tables 1.2−1.6, pp. 23−25 of the book. Copies are included here for completeness.

1.1. Solutions to the End of Chapter Problems

Problem 1.1. A body of constant mass m is acted on by force F which results in linear motion at constant velocity v. Show that the linear momentum mv is equal to the time integral of the force.

Solution:

$$\begin{aligned} F &= \text{time rate of change of linear momentum} \\ &= m\frac{dv}{dt} = \frac{d}{dt}(mv) \end{aligned}$$

Integrating both sides

$$\int F dt = mv \quad \text{if the constant of integration is zero}$$

Problem 1.2. A body of mass 100 kg initially rests on a ledge 25 m above the ground. It then falls freely to the ground under the influence of gravity. Air friction may be neglected. Gravitational constant $g = 9.81\,\text{m/s}^2$.

(a) What are initial and final values of the potential energy?
(b) What are the initial value and final value after impact of the kinetic energy?
(c) Calculate the instantaneous velocity at the mid-height of the fall. Hint: Use energy balance.

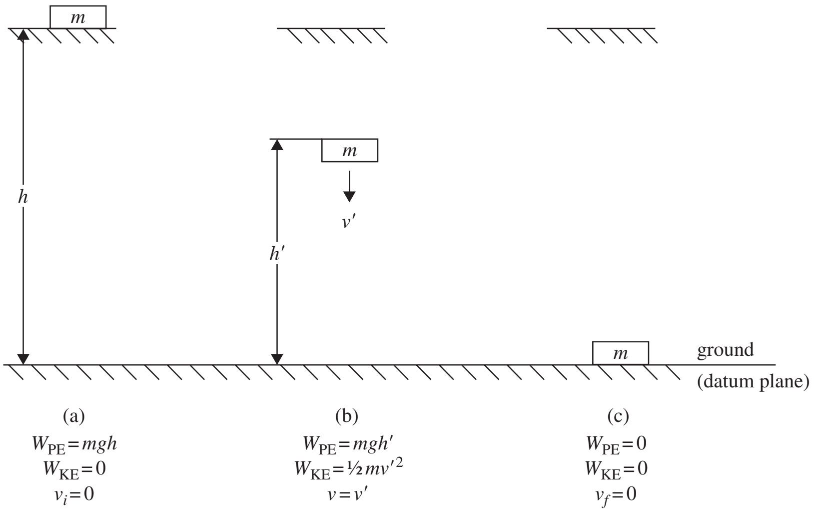

Fig. 1.2. Mass falling freely under gravity.

(d) Calculate the values of potential energy and kinetic energy at the mid-height of the fall.

Solution: (refer to Fig. 1.2)

(a) Initially $W_{\mathrm{PE}} = 100 \times 9.81 \times 25 = 24{,}525\,\mathrm{J}$
finally $W_{\mathrm{PE}} = 100 \times 9.81 \times 0 = 0$

(b) Initially $W_{\mathrm{KE}} = \frac{1}{2}\, mv_i^2 = 0$
finally $W_{\mathrm{KE}} = \frac{1}{2}\, mv_f^2 = 0$

(c) Equating the PE and the KE at the mid-height

$$mg\frac{h}{2} = \frac{1}{2}\, mv^2$$
$$v = \sqrt{gh} = \sqrt{9.81 \times 25} = 15.66\,\mathrm{m/s}$$

(d) $W_{\mathrm{PE}} = mgh = 100 \times 9.81 \times 12.5 = 12{,}262\,\mathrm{J}$
$W_{\mathrm{KE}} = \frac{1}{2}\, mv^2 = \frac{1}{2} \times 100 \times (15.66)^2 = 12{,}262\,\mathrm{J}$

Problem 1.3. A force of 100 N acts on a mass of 100 kg.

(a) What is the resulting linear acceleration?
(b) If the steady-state velocity is 10 m/s, what are the values of the kinetic energy and momentum?

Solution:

(a) $F = 100\,\text{N}, \quad m = 100\,\text{kg}, \quad v = 10\,\text{m/s}, \quad a = 100/100 = 1\,\text{m/s}^2$

(b) $W_{\text{KE}} = \frac{1}{2}\,mv^2 = \frac{1}{2} \times 100 \times 100 = 5000\,\text{J}$
$mv = 100 \times 10 = 1000\,\text{kg/s}$

Problem 1.4. A mass of 1 kg is rotated in a horizontal circle, at the end of a rigid tie-rod, with an angular velocity of 10 rad/s. If the radius of gyration is 0.5 m, what is the instantaneous linear velocity of the mass? Calculate the torque and angular acceleration if a force of 10 N is needed to maintain the rotation.

Solution:

$$m = 1\,\text{kg} \quad w = 10\,\text{rad/s} \quad r = 0.5\,\text{m} \quad F = 10\,\text{N}$$
$$v = wr = 10 \times 0.5 = 5\,\text{m/s}$$
$$T = Fr = 10 \times 0.5 = 5\,\text{Nm}$$

Angular accel $= \alpha = T/mr^2$ (Equation 1.14)

$$\alpha = \frac{5}{1 \times (0.5)^2} = \frac{5}{0.25} = 20\,\text{rad/s}^2$$

Problem 1.5. Calculate the moment of inertia of the rotating mass in Problem Qn. 1.4.

Solution:

$$\text{Polar moment of inertia} = J = mr^2$$
$$r = \text{radius of gyration} = 0.5\,\text{m}$$
$$J = mr^2 = 1 \times (0.5)^2 = 0.25\,\text{kg}^2$$

Problem 1.6. A mass of 1 kg rotates in a horizontal circle, at a radius 0.5 m about its fixed anchor point, with an angular velocity of 10 rad/s. What is the kinetic energy of the motion?

Solution:

From (1.17), $m = 1\,\text{kg}, \quad r = 0.5\,\text{m}, \quad w = 10\,\text{rad/s}$

$$W_{\text{KE}} = \frac{1}{2}\,Jw^2 = \frac{1}{2}\,mr^2w^2 = \frac{1}{2} \times 1 \times (0.5)^2(10)^2 = 12.5\,\text{J}$$

Problem 1.7. If a mass of 10 kg rotates around a circle of 1 m radius at 1800 rpm, what is its energy of motion?

Solution:

$$w = 1800\,\text{rpm} = \frac{1800}{60} \times 2\pi = 6\pi\,\text{rad/s}$$
$$r = 1\,\text{m} \quad m = 10\,\text{kg}$$
$$W_{\text{KE}} = \frac{1}{2}mr^2w^2 = \frac{1}{2} \times 10 \times 1^2 \times (6\pi)^2 = 1776\,\text{J}$$

Problem 1.8. An imperial gallon of water is uniformly heated so that its temperature increases by 20°C. What is the rise of its heat energy content?

Solution: 1 imperial gallon = 4.55 litres = 4550 cm^3 = 4.55 kg
From (1.19),

$$Q = m \times SH \times \text{temp rise} = 4550 \times 1 \times 20 = 91{,}000\,\text{cal.}$$

From (1.20),

$$W = 4.2\ Q = 4.2 \times 91,000 = 382.2\,\text{kJ}$$

Problem 1.9. Two equal masses of water are mixed in a container. What is the final temperature of the mixture if (a) the two initial temperatures T_{in} are equal, (b) one mass has an initial temperature twice that of the other?

Solution:

(a) Final temp T_f = initial temp T_{in}
(b) From Example 1.4,

$$T_f = \frac{m_1SH_1T_1 + m_2SH_2T_2}{m_1SH_1 + m_2SH_2}$$

Since

$$m_1 = m_2, \quad SH_1 = SH_2, \quad T_1 = 2T_2, \quad T_f = \frac{T_1 + 2T_1}{1+1} = \frac{3}{2}T_{\text{in}}$$

Problem 1.10. In the UK the original steam engines designed by Watt and Newcomen used reservoir temperatures of 100°C and 10°C. What was the maximum theoretical efficiency?

Solution:

$$10°\text{C} = 283\,\text{K}, \quad 100°\text{C} = 373\,\text{K}$$

From (1.24),

$$\eta_{\text{CARN}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{283}{373} = 1 - 0.756 = 24.1\%$$

Problem 1.11. A heat engine operates with a high temperature source of 900 K and initial heat energy of 500 MJ. Its low temperature sink operates at 300 K. The maximum realisable efficiency is one-half the value of the theoretical maximum value.

(a) Calculate the maximum working value of the efficiency.
(b) What is the maximum work output?

Solution:

(a)

$$T_{\text{H}} = 900\,\text{K}, \quad T_{\text{L}} = 300\,\text{K}, \quad W = 500\,\text{MJ}$$
$$\eta_{\text{CARN}} = 1 - \frac{300}{900} = 1 - 0.333 = 0.667$$
$$\eta_{\text{realisable}} = \eta_{\text{CARN}}/2 = 0.333$$

(b)

$$\text{Work output} = \text{work input} \times \text{working efficiency}$$
$$= 500 \times 0.333 = 166.5\,\text{MJ}$$

Problem 1.12. For the purpose of converting heat energy into useful work from an ambient temperature of 100°C, is it better to have one heat source Q of temperature 400°C or two equal sources Q of temperature 200°C?

Solution: One source Q at temp 400°C or two sources Q at temp. of 200°C?

$$\eta_1 = 1 - \frac{373}{673} = 1 - 0.55 = 0.45$$

Heat energy $= \eta\, Q = 0.45\,Q$ with one source

$$\eta_2 = 1 - \frac{373}{473} = 1 - 0.79 = 0.21$$

Heat energy $= 2 \times 0.21\,Q = 0.42\,Q$ with two sources.
$\therefore$ Better to use the single source.

Problem 1.13. A heating boiler has a full-load working efficiency of 65%. It is used to heat a building from the outside temperature of 35°F to 68°F. What is the total thermal efficiency?

Solution: Using the temperature conversions from Table 1.6,

$$35°\text{F} \equiv \frac{5}{9}(35-32) = \frac{15}{9} = 1.67°\text{C} = 274.7\,\text{K}$$
$$68°\text{F} \equiv \frac{5}{9}(68-32) = \frac{5}{9} \times 36 = 20°\text{C} = 293\,\text{K}$$
$$\eta_{\text{CARN}} = 1 - \frac{274.7}{293} = 1 - 0.94 = 0.06 = 6\%$$
$$\eta_{\text{pract}} = 0.65 \times 0.06 = 0.39 \equiv 39\%$$

Problem 1.14. For the steam boiler–electricity generator system of Fig. 1.5

(a) Show that the first law of thermodynamics is satisfied.
(b) Calculate the efficiency of the turbine.
(c) What information is needed in order to calculate the energy discharged through the chimney?
(d) Calculate the efficiency of the generator.

Solution:

(a) From Fig. 1.5,

$$Q_{\text{H}} = 11.194 \text{ MBTU}, \quad Q_{\text{L}} = 5.933\,\text{MBTU}, \quad W = 5.261 \text{ MBTU},$$
$$\therefore Q_{\text{H}} - Q_{\text{L}} = W \quad \text{verifying (1.22)}$$

(b) $\eta_{\text{turb.}} = \dfrac{5.261}{11.194} = 47\%$

(c) It is necessary to determine the energy content of 1000 pounds of solid fuel and deduct 12.72 MBTU.

(d) $\eta_{\text{gen}} = \dfrac{5.209}{5.261} = 99\%$

This is the order of efficiency that you would expect for the continuous working of a large turbo-generator set rated at (say) 600 MW.

Problem 1.15. A Carnot engine has a low temperature sink of 10°C and a maximum theoretical efficiency of 38%. By how much does the temperature of the high temperature source need to increase in order to raise the efficiency to 50%?

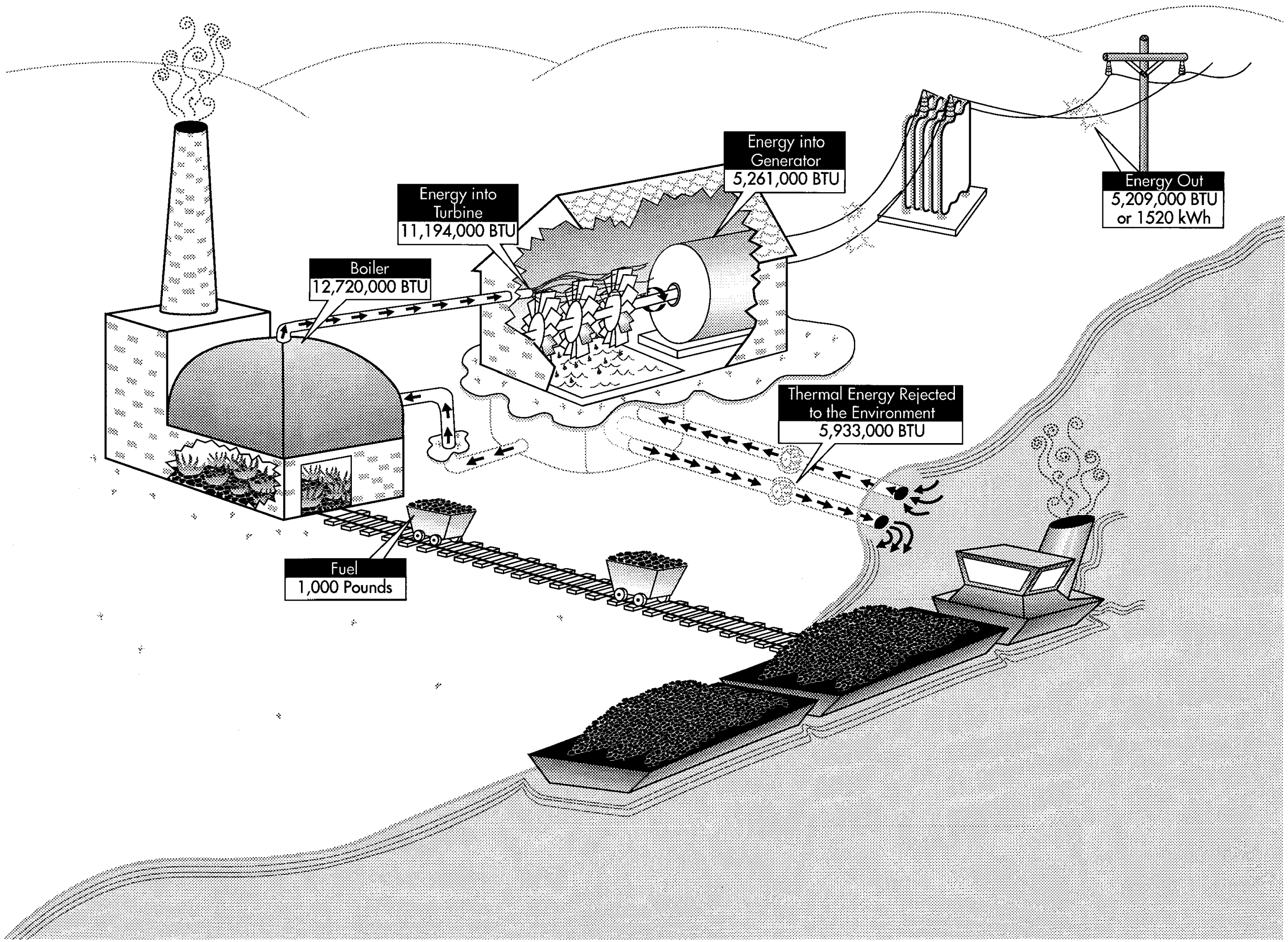

Fig. 1.5. Basic form of a heat–work system for electricity generation (based on an idea in Dorf [2]).

Solution:

$$T_{\mathrm{L}} = 10^{\circ}\mathrm{C} = 283\,\mathrm{K}$$

in general

$$\eta_{\mathrm{CARN}} = 1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}$$

Initially

$$0.38 = 1 - \frac{283}{T_{\mathrm{H}}} \quad \text{or} \quad T_{\mathrm{H}} = 456.5\,\mathrm{K}.$$

Finally

$$0.5 = 1 - \frac{283}{T_{\mathrm{HL}}} \quad \text{or} \quad T_{\mathrm{H}} = 566\,\mathrm{K}$$

Temp rise $= 566 - 456.5 = 109.5^{\circ}\mathrm{C}$

Problem 1.16. Explain what happens to the power input to a refrigerator if its door is left open in a warm room.

Solution: With the refrigerator door open the cooling mechanism attempts to refrigerate the room. It works continuously, at full power. This is obviously a waste of electrical input energy. The refrigerator is designed to cool its design volume (the interior of the fridge) and it operates at reduced efficiency in any other application.

Problem 1.17. A high temperature fluid contains 1000 MJ of energy at 600°C. This fluid powers a mechanical converter of Carnot efficiency 30%.

(a) What is the temperature of the sink fluid?
(b) What is the change of entropy?

Solution:

$$T_{\mathrm{H}} = 600^{\circ}\mathrm{C} = 873\,\mathrm{K}$$

If η_{CARN} is the maximum theoretical efficiency, in this case 30%, then

(a) $0.3 = 1 - \dfrac{T_{\mathrm{L}}}{873}$ and $T_{\mathrm{L}} = 611\,\mathrm{K}$

(b) From (1.31),

$$\Delta S = \frac{700}{611} - \frac{1000}{873} = 1.1457 - 1.1455 = +0.0002\,\mathrm{J/K}$$

Problem 1.18. What are the centigrade (Celsius) equivalents of the following temperatures in degrees Fahrenheit? (a) 212°F, (b) 100°F, (c) 32°F, (d) 0°F.

Solution: From Table 1.6,

$$°\text{C} = \frac{5}{9}(°\text{F} - 32)$$

(a) $\frac{5}{9}(212 - 32) = \frac{5}{9} \times 180 = 100°\text{C}$

(b) $\frac{5}{9}(100 - 32) = \frac{5}{9} \times 68 = 37.8°\text{C}$

(c) $\frac{5}{9}(32 - 32) = 0°\text{C}$

(d) $\frac{5}{9}(0 - 32) = -\frac{5}{9} \times 32 = -17.8°\text{C}$

Problem 1.19. What are the Fahrenheit equivalents of the following temperatures in degrees centigrade? (a) 212°C, (b) 100°C, (c) 32°C, (d) 0°C.

Solution: From Table 1.6,

$$°\text{F} = \frac{9}{5}°\text{C} + 32$$

(a) $\frac{9}{5} \times 212 + 32 = 381.6 + 32 = 413.6°\text{F}$

(b) $\frac{9}{5} \times 100 + 32 = 180 + 32 = 212°\text{F}$

(c) $\frac{9}{5} \times 32 + 32 = 57.6 + 32 = 89.6°\text{F}$

(d) $\frac{9}{5} \times 0 + 32 = 32°\text{F}$

Problem 1.20. At what value of temperature is the temperature reading in degrees centigrade equal to the reading in degrees Fahrenheit?

Solution: Let the common temp $= X$

Using the conversion of Problem 1.19 above

$$X = \frac{9}{5}X + 32, \quad -32 = \frac{9}{5}X - X = \frac{4}{5}X, \quad X = \frac{5}{4}(-32) = -40°$$

Therefore $-40°\text{C} \equiv -40°\text{F}$

Problem 1.21. What is the centigrade equivalent of 75.8°F?

Solution: From Table 1.6,

$$^\circ\mathrm{C} = \frac{5}{9}(^\circ\mathrm{F} - 32)$$

in this case

$$^\circ\mathrm{C} = \frac{5}{9}(75.8 - 32) = \frac{5}{9} \times 43.8 = 24.33^\circ\mathrm{C}$$

Problem 1.22. What is the Fahrenheit equivalent of 19.6°C?

Solution: From Table 1.6,

$$^\circ\mathrm{F} = \frac{9}{5}{}^\circ\mathrm{C} + 32$$

in this case

$$^\circ\mathrm{F} = \frac{9}{5} \times 19.6 + 32 = 35.3 + 32 = 67.3^\circ\mathrm{F}$$

Problem 1.23. A modern electric power station has a full load of 2000 MW. What are the equivalent values in (a) horsepower, (b) joules/second, (c) kilowatts, (d) footpounds/second?

Solution: Power generation station rated at 2000 MW.

Using the conversion factors from Table 1.5,

(a) $1\,\mathrm{HP} \equiv 746\,\mathrm{W}$

$$\therefore 2000\,\mathrm{MW} \equiv \frac{2000 \times 10^6}{746} = 2.68 \times 10^6\,\mathrm{HP}$$

(b) $1\,\mathrm{J/s} \equiv 1\,\mathrm{W}$

$$\therefore 2000\,\mathrm{MW} = 2000 \times 10^6 = 2000\,\mathrm{MJ/s}$$

(c) $1\,\mathrm{kW} = 1000\,\mathrm{W} = \frac{1}{10}\,\mathrm{MW}$

$$\therefore 2000\,\mathrm{MW} = 2000 \times 1000 = 2 \times 10^6\,\mathrm{kW}$$

(d) $1\,\mathrm{HP} \equiv 746\,\mathrm{W} \equiv 550\,\mathrm{ft.\,lb/s}$

$$\therefore 2000\,\mathrm{MW} = \frac{2 \times 10^9}{746} \times 550 = 1470\,\mathrm{ft.lbs/s}$$

Problem 1.24. A solar water heating panel has a thermal energy rating of 50 MJ. What is the rating in kilowatt hours?

Solution: From Table 1.6,

$$1\,\text{kWh} \equiv 3.6 \times 10^6\,\text{J}$$

$$\therefore 50\,\text{MJ} = 50 \times 10^6\,\text{J} \equiv \frac{50}{3.6} = 13.9\,\text{kWh}$$

Problem 1.25. The large wind turbine at Burger Hill, Orkney, Scotland, is rated at 3 MW. What is the equivalent rating in horsepower?

Solution: From Table 1.6,

$$1\,\text{HP} \equiv 746\,\text{W}$$

$$\therefore 3\,\text{MW} = \frac{3 \times 10^6}{746} = 4021.5\,\text{HP}$$

1.2. Additional Problems with Solutions

Problem 1.26. What is the centigrade equivalent of 78.5°F?

Solution: From Table 1.6,

$$^\circ\text{C} = \frac{5}{9}(^\circ\text{F} - 32)$$

With 78.5°F,

$$^\circ\text{C} = \frac{5}{9}(78.5 - 32) = \frac{5}{9}(46.5) = 28.83^\circ\text{C}$$

Problem 1.27. What is the Fahrenheit equivalent of 28.5°C?

Solution: From Table 1.6,

$$^\circ\text{F} = \frac{9}{5}{}^\circ\text{C} + 32$$

With 28.5°C,

$$^\circ\text{F} = \frac{9}{5} \times 28.5 + 32 = 51.3 + 32 = 83.3^\circ\text{F}$$

Problem 1.28. The largest wind turbine generators now in operation are rated at 5 MW. What is the equivalent rating in horsepower?

Solution:

$$1\,\text{HP} \equiv 746\,\text{W}$$

$$5\,\text{MW} = 5 \times 10^6\,\text{W} \equiv \frac{5 \times 10^6}{746} \equiv 6702.4\,\text{HP}$$

Problem 1.29. Express the thermal energy rating 75 MJ in kilowatt hours.

Solution: From Table 1.5,

$$1\,\text{kWh} \equiv 3.6 \times 10^6\,\text{J}$$

$$75\,\text{MJ} \equiv \frac{75}{3.6} \equiv 20.83\,\text{kWh}$$

Problem 1.30. What is 100 horsepower expressed in kilowatts?

Solution: From Table 1.5,

$$1\,\text{HP} \equiv 746\,\text{W}$$
$$\therefore 100\,\text{HP} \equiv 74{,}600\,\text{W} \equiv 74.6\,\text{kW}$$

Problem 1.31. What is the equivalent of one kilowatt hour in (a) joules, (b) British Thermal Units?

Solution: From Table 1.5,

(a) $1\,\text{kWh} \equiv 3.6 \times 10^6\,\text{J} = 3.6\,\text{MJ}$
(b) $1\,\text{kWh} \equiv 3.412 \times 10^3\ \text{BTU} = 3.412\,\text{kBTU}$

Problem 1.32. A modern turbo-generator is rated at 600 MW.
What is the equivalent rating in (a) horsepower, (b) joules/s, (c) kilowatts?

Solution: From Table 1.5,

(a) $600\,\text{MW} \equiv 600 \times 10^6 \times 0.001341 \equiv 600 \times 1341 = 804{,}600\,\text{HP}$

(b) $600\,\text{MW} = 600 \times 10^6\,\text{W} \equiv 600 \times 10^6\,\text{J/s}$

(c) $600\,\text{MW} = \dfrac{600 \times 10^6}{10^3} = 600{,}000\,\text{kW}$

1.3. Units and Conversion Factors

Table 1.2. The International System of Units (S.I.).

Property	Unit	Symbol
Basic		
Length	metre (UK), meter (US)	m
Mass	kilogramme	kg
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Derived		
Velocity	metre per second	v (m/s)
Area	square metre	A (m^2)
Force	newton	F (kgm/s^2)
Energy (Work)	joule (newton-metre)	W (J *or* Nm)
Power	watt	P (J/s)

Table 1.3. Multiples and sub-multiples in S.I. units.

Unit	Symbol	Value
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

Table 1.4. Conversion factors.

Length	
1 millimetre (mm)	0.0393701 inch (in)
1 metre (m)	3.28084 feet (ft)
Area	
1 square centimetre (cm^2)	0.155000 in^2
1 square metre (m^2)	10.7639 ft^2
1 hectare = $10^4 m^2$	2.4710 acres
Volume	
1 cubic centimetre (cm^3)	0.0610237 in^3
1 cubic metre (m^3)	35.31477 ft^3
1 litre (1) (1000 cm^3)	1.75985 UK pints
1 imperial gallon (UK)	4.54596 litres
1 US gallon	3.78531 litres
1 barrel = 42 US gallons = 34.97 UK gallons = 159.00 litres	
Weight	
1 kilogramme (kg)	2.20462 lb
1 tonne (10^3 kg)	0.9984207 ton (UK)
1 ton (UK) or statute or long ton = 1.120 short tons	
Force	
1 newton (N)	0.2248 lb force
Pressure	
1 pascal (Pa)	1 N/m^2
1 bar = 10^5 Pa	14.50 lbf/in^2
1 lbf/in^2 (one pound per square inch or psi)	6.89476 kPa
Atmospheric pressure = 14.70 lbf in^{-2}	101.325 kPa
Velocity	
1 mile per hour (mph)	0.447 m/s
1 kilometre per hour (kph)	0.278 m/s

Table 1.5. Conversion factors in power, heat and energy.

	Unit	Equivalents
Power		
	1 watt (W)	1 Joule/sec (J/s) = 0.001341 HP
	1 kilowatt (kW)	1000 W = 1.34 HP
	1 horsepower (HP)	745.7 W = 550 ft lb/sec
Power density		
	1 W/m^2	3.6 $kJ/m^2/h$ = 0.317 $BTU/ft^2/h$
Heat energy		
	1 calorie (cal)	4.1868 J
	1 British thermal unit (BTU)	1055.06 J = 778.169 ft lb
	1 therm	10^5 BTU = 29.3 kWh = 1.05506×10^8 J
Heat energy density		
	1 $kcal/m^2$	0.3687 BTU/ft^2 = 1.163 Wh/m^2
	1 BTU/ft^3	3.726×10^4 J/m^2
	1 Langley	1 cal/cm^2 = 41868 J/m^2
Energy		
	1 Joule	1 watt-second (Ws)
	1 electron volt (eV)	1.602×10^{-19} J
	1 kilowatt hour (kWh)	3.6×10^6 J = 3.412×10^3 BTU

Table 1.6. Scales of temperature.

The centigrade (Celsius) scale of temperature has 100 degree units between the freezing point 0°C and boiling point 100°C of water at standard pressure.

The Fahrenheit scale has 180 degree units between the freezing point 32°F and boiling point 212°F of water. Therefore

$$100°\text{C} \equiv 180°\text{F}$$

and

$$°\text{C} = 5/9\ (°\text{F} - 32)$$

$$°\text{F} = 9/5\ °\text{C} + 32$$

The Kelvin scale of temperature is measured from absolute zero −273.15°C, usually rounded to −273°C. Therefore, for temperatures greater than zero degrees centigrade,

$$\text{K} = °\text{C} + 273$$