Chapter 1

An Early Life History and Career of Sir James Lighthill

"... by summing up the argument of my lecture in a single sentence: It needs categorically to be reaffirmed that the continuum mechanics of a fluid innocent of electric current has as vital and exciting a present and future as any other branch of physical science."

"... as Sir Cyril Hinshelwood has observed ... fluid dynamicists were divided into hydraulic engineers who observed things that could not be explained and mathematicians who explained things that could not be observed."

James Lighthill

The old city of Paris has many great traditions. It is the capital and largest city of France. It is one of the most beautiful cities in the world. Three of the most famous Parisian landmarks are the twelfth century Notre Dame de Paris on the Île de la Cité, the nineteenth century Eiffel tower and the Napoleonic Arc de Triumphe. The 1063 feet tall Eiffel Tower is the most recognized symbol of Paris and it is located on the bank of the River Seine in north-central France. At night, floodlights shine on Paris' lovely parks, gardens, palaces and monuments. The gleaming beauty of Paris has given it the nickname *City of Light*. It has a reputation as a "romantic city". Paris is widely recognized as one of the world's global cities. It is a leading global cultural, educational, business and political centre and is renowned for its classical architecture as well as its unique role as a major international influence in modern fashion, gastronomy, museums and arts.

More importantly, Michael James Lighthill was born on January 23, 1924 in the Rue Puccini, Paris. His father, Earnest Balzar Lighthill, was mining engineer who worked all over the world but came to England when he retired in 1927 at the age of 59. His mother, Marjorie, was a daughter of Yorkshire engineer, L. W. Holmes and she was 18 years younger than her husband. James had a brother, Olaf, seventeen years older and a sister, Patricia, nine years older than himself. Obviously, James was a new member of a middle class family and grew in his family under the dominant influence of parental love and care.

James was a special and precocious child, showing a clear disposition to originality and independence and early evidence not only of his mathematical talent and of his photographic memory, but also in many other areas including language, music and chess. At the age of ten, James beats his father in a chess game, even though his father was a very good chess player.

James spent early years at private schools and won a Scholarship to Winchester College that was one of the most famous and ancient public schools in England. It was a remarkable coincidence that Freeman Dyson, a twelve-year old scholar at Winchester, was James' classmate at Winchester. Freeman Dyson spent many years and is still at the Institute for Advanced Study in Princeton. Professor Dyson is one of the brilliant mathematical physicists of the twentieth century. James and Freeman became very close friends and spent considerably amount of time together to study mathematics. At the age of fourteen, they spent considerable amount of time to study *Principia Mathematica* (1910-1913) by A. N. Whitehead (1861-1947) and Bertrand Russell (1872-1970) who published three monumental volumes of the *Principia* containing a completely rigorous development of the foundations of mathematics. At the same time, in order to learn mathematical analysis, James read the three-volume *Cours d'analyse de l'Ecole polytechnique* by Camille Jordan (1909). Undoubtedly, this study was a most profound basis for Lighthill's subsequent mastery of mathematical analysis and its applications.

After a rigorous education at Winchester College with an outstanding record, at the age of fifteen, James and Freeman were both awarded prestigious scholarships to Trinity College, Cambridge to pursue a B.A. degree in Mathematics. But the college would not allow them to join until they were seventeen years old. After spending two years of study in more mathematics, they entered the Trinity College in 1941 and began a new life as undergraduate students. Since they had already learned sufficient undergraduate material in mathematics, they chose to attend only the lectures for Part III of the Mathematical Tripos almost entirely in pure mathematics intended for graduate students. James was particularly influenced by the presence of famous pure mathematicians including G. H. Hardy (1877-1947), J. E.

Littlewood (1885-1977) and A. S. Besicovitch (1891-1970) in Cambridge at that time. Lighthill's Cambridge supervisor was Professor Besicovitch who became worried about James' allegiance to pure mathematics that would be completely lost if he moves out from Cambridge. Originally from St. Petersburg, Besicovitch was a renowned pure mathematician for his major contributions to the theory of almost periodic functions and other areas of function theory, and especially, for his pioneering research work in geometric measure theory, where he proved many of the fundamental results. Since there was then little incentive at Cambridge to study applied mathematics, Besicovitch reluctantly recommended James to join Sydney Goldstein's (1903-1989) group at the British National Physical Laboratory (NPL), and wrote to Goldstein begging him not to ruin James. The Mathematical Tripos in those days was a very competitive examination for which many of the candidates were coached personally. Candidates for this examination were put into the first, second, and third class according to the marks obtained and were also ordered within each class. Candidates placed into the top class 1 are called '*Wranglers*'. After two years of study, James and Freeman took both Part II and Part III of the Mathematical Tripos Examination. Not surprisingly, they obtained a first class in the former and a Distinction in the latter.

After successfully completing his B.A. degree from Cambridge in two years, James joined the British National Physical Laboratory as Junior Scientific Officer at Aerodynamics Division in 1943 and soon became a Senior Scientific Officer at NPL. Before he became twenty in 1945, James published his first research papers on two-dimensional supersonic aerofoil theory in the Aeronautical Research Committee Report and Memorandum (ARCRM). He continued his research at NPL and then published several other papers on airfoil theory in ARCRM in 1944 and 1945. His dream of becoming a successful applied mathematician has led him to participate and excel in many other diverse activities at NPL, and to become a well-rounded and knowledgeable research scientist with highest possible standards.

In 1945, at the age of twenty one, James married his undergraduate classmate, Nancy Dumaresq who was studying mathematics at Newnham College in Cambridge. Nancy also got a job at the Royal Aircraft Establishment (RAE) at Farnborough. James was asked to go to the NPL at Teddington and to conduct his research with the distinguished fluid dynamicist, Sydney Goldstein (1903-1989) who convinced James Lighthill to pursue research in fluid mechanics as this subject has a wide variety of challenging research problems. Because of his outstanding research work at NPL, James was awarded major research fellowship at Trinity College, Cambridge in 1945. During his stay for a year at his ‘Alma Mater’, Lighthill was fully influenced by Sydney Goldstein and G. I. Taylor (1886-1972) to continue his research on challenging problems in fluid mechanics. G. I. Taylor has already become one of the great physical scientists of the twentieth century for his numerous notable theoretical and experimental work in fluid and solid mechanics with applications to oceanography, meteorology, mechanical, civil and chemical engineering, hydraulics and materials science. I was fortunate to see this distinguished scientist and great man many times at the Department of Applied Mathematics and Theoretical Physics of the University of Cambridge during my stay in Cambridge in 1968-1969. The awarding of that prestigious fellowship to James Lighthill combined with the encouragement of Goldstein and Taylor launched a remarkable research career and led directly to lifelong association with fluid mechanics.

In 1946 Sydney Goldstein joined the University of Manchester as the Beyer Chair of Applied Mathematics succeeding Sir Horace Lamb, another renowned applied mathematician and fluid dynamicist, and persuaded James Lighthill to go to there as a Senior Lecturer. Almost simultaneously, Max Newman (1897-1984), a distinguished topologist, was appointed to the Fielden Professorship of Pure Mathematics of the University of Manchester. Sir John Stopford, then Vice Chancellor of Manchester, convinced the Senate and Academic Council that Sydney Goldstein and Max Newman would create at Manchester a dynamic and internationally renowned new Department of Mathematics, interacting admirably with other units of the University. After spending a year in Cambridge, Lighthill joined the University of Manchester as Senior Lecturer in Mathematics at the age of twenty two. Within a short period of four years, he succeeded Goldstein in the Beyer Professor of Applied Mathematics at Manchester at the age of twenty six. This has been almost an extraordinary accomplishment in the United Kingdom in those days. It was, indeed, with a definite commitment in mind, Lighthill arrived at Manchester in 1945. It became the starting point for his forceful participation in moving Manchester forward to still greater levels of excellence in applied mathematics and fluid mechanics. During his thirteen years that were spent helping to the University of Manchester which was outstandingly strong in applied mathematics and fluid mechanics, Lighthill pursued also many other interests. In particular, he was actively involved in maintaining and still further developing those great Manchester traditions that were initiated by Sydney Goldstein and Max Newman. He continued his active interests in interdisciplinary research with engineering scientists and expanded his interaction with other parts of the University. In addition, the Fluid Motion Laboratory became an active and productive Department of Mechanics of Fluids, still working in close cooperation and collaboration with mathematics faculty and post graduate students. His over a decade long stay at Manchester was very productive and rewarding in his life in terms of research, publications and supervision of doctoral students—many of them are now well known in applied mathematics. Highly respected by his applied mathematics peers, he was instrumental in organizing a strong research group in fluid mechanics and applied mathematics in England. Above all, he created an enduring cordial atmosphere of mutual support and encouragement for fluid dynamics research at the highest level. In their biographical memoir, both David Crighton (1942-2000) and T. J. Pedley recognized Lighthill's remarkable success as a teacher-scholar in Manchester as being due to his profound interest in applied mathematics and fluid mechanics and his faculty for inspiring his colleagues, and especially the best research graduate students, with some of his own enthusiasm.

In 1952 Lighthill was elected Fellow of Royal Society of London at the age of twenty nine for his pioneering research on sound generated aerodynamically. He was awarded the Bronze Medal of the Royal Aeronautical Society for his outstanding contributions to aerodynamics in 1955.

During his happy and extremely productive period of 13 years in Manchester, Lighthill was deeply involved in developing a modern pure and applied mathematics research center in collaboration with several eminent pure mathematicians including Max Newman and others. He served as research supervisor of many Ph.D students and other research scholars. Subsequently, almost all of his students and research associates became famous and eminent applied mathematicians for their own original research contributions. Usually, Lighthill did not wish to write joint papers with his doctoral and post doctoral students. Amazingly, Lighthill produced a large number of long research papers and two major books by himself during his tenure at the University of Manchester. In addition, he delivered many research expository and survey lectures as well as famous memorial lectures by invitation from many national and international professional and scientific organizations. Indeed, almost all of his research papers and lectures were loaded with new ideas and results, and represent pioneering contributions to a wide variety of different areas of fluid mechanics and applied mathematics. In 1999, D. G. Crighton described the long range impact of Lighthill's research as: "... worked extensively on gas dynamics, including effects of shock and blast waves. He also launched two major new fields in fluid mechanics."

The first of these new fields was aeroacoustics which proved to be vital importance in the reduction of noise from jet engines. In his research on aeroacoustics, Lighthill discovered his famous *eighth-power law* which states that the acoustic power radiated by a jet aircraft is directly proportional to the eighth power of the jet speed. In fact, Lighthill's theory and his eighth-power law provided a revolutionary impact on the modern mechanics, in general, and on aircraft industry, in particular. The second new field introduced by Lighthill was modern nonlinear acoustics in which he described propagation of shock waves, flood waves in rivers and traffic flow on highways. His 1956 classic 101-page long chapter on "Viscosity Effects in Sound Waves of Finite Amplitude" written in honor of the 70th birthday of great fluid dynamist and physical scientist Sir G. I. Taylor can be considered as the beginning of nonlinear acoustics. On the other hand, in order to extend the classical work of Sir Geoffrey Taylor on shock waves, Lighthill developed a more general and complete theory of plane shock wave formation with conflicting effects of convection (nonlinear) on one hand, and of diffusion and relaxation on the other, where these effects were allowed to first approximation. However, sound waves of finite amplitude with effects of relaxation were investigated by Bethe and Teller (1941). More general theory of the effect of relaxation on general steady flows had been fully treated by Kantrowitz (1946) and Gunn (1946). In his research on this field, Lighthill first initiated a new and modern development of nonlinear acoustic wave propagation and unified the research work of other eminent applied mathematicians including J. M. Burgers (1895-1981), J. D. Cole (1925-1999), H. Hopf (1894-1971), G. I. Taylor, and G. B. Whitham.

According to Professor T. J. Pedley (2001), Lighthill's 13-year stay in Manchester represents his "golden years" as this period was full of intensive intellectual activity during which he has not only published a large volume of research work and two major books, but also produced many future national and international leaders in applied mathematics. In order to promote research interest in a wide range of fluid mechanics, Lighthill initiated a series of annual conferences called the *British Theoretical Mechanics Colloquia (BTMC)* which was first held in Manchester in 1959. These colloquia became the national forums for *British Applied Mathematics Colloquia* in a short period of time. Lighthill delivered many invited lectures at the Annual Conference of BTMC. I myself participated in two BTMC Conferences at Southampton in 1966 and at Oxford in 1968 and enjoyed Lighthill's invited lectures on rotating fluids and dynamics of oceans at these conferences. In appreciation for his remarkable contributions, the Senate and Council of the University of Manchester adopted the valedictory resolution in February 1960 which states:

"Those of us who knew Lighthill in his younger days remember a man of almost frightening brilliance, a prickly mathematician intolerant of any argument which lacked precision and rigor, and devastating in his verbal attack on its author. His standards never changed: but his attitude towards people did. Later he would offer gentle criticism and advice and the most painstaking guidance; and his students, whether the bright or the weakest, received the same patient interest and encouragement."

Lighthill left Manchester in 1959 to take the extremely prestigious position of Director of the Royal Aircraft Establishment (RAE) at Farnborough. His main leadership role included managing the work of the fourteen hundred scientists and engineers, and the eight thousand RAE staff. Despite his large administrative duties and responsibilities as Director, Lighthill was extremely active and productive in his own research, publications and presentations. Indeed, he continued to publish papers and books at an unprecedented rate. For example, he published his lecture on Fluid Dynamics as a Branch of Physics in *Physics Today* in 1962. This lecture was presented at a well attended Banquet Meeting of the Fluid Dynamics Division of the American Physical Society in November 1960. He began his lecture with the following interesting statements: "... why fluid dynamist remained almost completely unabsorbed by physics until considerably less than a century ago, and to note how the process of absorption has taken place only gradually since then, initially as a result of the work of Rayleigh and Prandtl and von Kármán and G. I. Taylor and the rest, and how it has proceeded with accelerated speed over the last thirteen years, greatly as a consequence of the work of members of Fluid Dynamics Division of the American Physical Society.

I'd like to remark again that the great physicists from Newton onwards were notable for unceasingly comparing theory and experiment. ... you may well think unreasonable to criticize Sir Isaac, bearing in mind that he was breaking completely new ground in everything he did, and in particular in his studies of fluid dynamics ..."

"Similar criticisms, couched in the vein of comparing work in this field, can be made of the approaches of Euler, Lagrange, Cauchy and Kelvin to theoretical fluid dynamics. The mathematical reasoning was on a notably high level, but the extent of contact with real fluids was kept to a minimum, and direct attempts at comparison with experiment to a large extend avoided. ...”

“Writers at that time compared what they called the “older” and “newer” theories, namely Newtonian and potential flow theories, and came to the conclusion that the Newtonian must be preferred because it avoided the d’Alembert paradox. This ignored the fact that fluid motion which it predicted different fundamentally from observation.....”

“Looking back in the light of our knowledge that viscous forces, though small, play a crucial part in determining the flow about solid bodies, it is worth looking closely at the nineteenth-century work on viscous flow to see how this point was missed. A certain part of this work must be admitted to have been of outstanding value, notably the experiments on resistance in capillary tubes by Poiseuille and Hager, stimulated by physiology, and Stokes’s great paper of 1851 on the effect of viscosity on the motion of pendulums, stimulated by the problems of gravity surveying with pendulums in the rather low vacuum conditions which could then be achieved. All these pieces of work obtained excellent agreement with experiment using the condition of zero relative velocity of fluid at a solid surface, which Stokes showed was also to be expected on physical grounds.”

“In this field of external aerodynamics, even the appearance of Prandtl’s great paper of 1904, which contained all the essentials of the solution to the mystery, was by no means immediately effective in increasing understanding of the physics of the subject among the abstract mathematicians or the empirical engineers. Books do still appear which ascribe flow separation to causes which really are those underlying cavitation! Nevertheless, by the end of the twenties there had been enough university centers of instruction in the physics of fluid motion, notably Göttingen and Caltech, and enough handbooks of the subject like those of Durand, Goldstein, and the German compilations, to have created a recognized and adequately manned discipline which comprehended both theory and experiment.”

“I could also mention some other examples to emphasize that it was not only *external* aerodynamics which was slow to be integrated within physics. In the general area of vibrations and waves the intimate binding together of theory and experiment was made rather earlier, largely by Rayleigh, whose *Theory of Sound* is, justly, still one of the best-sellers of science. It is interesting, though, that not till well on into this century was it discovered that the mathematical theory of the attenuation of sound by viscosity and heat conduction gave results orders of magnitude too low, actually owing to the time lag in molecular vibrational energy exchange, and it was about the same time that the physical nature of a shock wave first became clear, while the application of surface wave theory to experimental oceanography had to wait for the nineteen-forties."

"Nevertheless, at the time when this Fluid Dynamics Division was formed, a clearly defined science had grown up on sound physical premises, with a wide range of applications in aeronautics, naval architecture, acoustics, meteorology, oceanography, mechanical engineering and chemical and explosives engineering. The basic physical premises were a continuum fluid and the simple laws of viscosity and heat condition. Ideas derivative from these included boundary layers, vortices, turbulence, sound waves, Mach lines and shock waves, actuator disks, free and forced convection, deep and shallow water waves, phase and group velocity, secondary flow, geostrophic and thermal winds. The consistency of inclusion within physics of fluid dynamics at this period is well shown by the success with which Landau and Lifschitz were able soon after this time to include in their nine-volume course on theoretical physics a volume on fluid mechanics, which recently has been translated into English...."

"Next, we know that liquid helium below its lambda point exhibits a fluid dynamics considerably different from those to which we are accustomed, and a quantum fluid dynamics has been worked out which indicates that in certain circumstances a proportion of the fluid mass is capable of motion but unable to carry entropy or vorticity. This discovery explained a whole range of phenomena, but there are others, beyond, where the low temperature physicists are now postulating a peculiar kind of turbulence in the superfluid...."

"This symposium has reminded us of the continued fascination and inexhaustibility of the study of water waves, and of low-speed aerodynamics. I was interested to find that at the wind tunnels of the National Aeronautics and Space Administration, as well as at those of the Royal Aircraft Establishment, it is the low-speed ones, the good old low-Mach-number wind tunnels, that are absolutely crammed with work!..."

His concluding statement of this stimulating lecture is a delight to read: "I could go on much longer, but I will just conclude by thanking you again for inviting me here, and by summing up the argument of my lecture in a single sentence: It needs categorically to be reaffirmed that the continuum mechanics of a fluid innocent of electric current has as vital and exciting a present and future as any other branch of physical science."

He also published his two major chapters on laminar boundary layer theory in the famous volume in *Laminar Boundary Layers* edited by L.

Rosenhead in 1963. At the same time, he established a new collaboration with the neighboring Institute of Aviation Medicine and created a Department of Space Research at the RAE. Even though he was not less successful obtaining funding for original research in aerodynamics in general, Lighthill gradually realized the major weakness of RAE, especially, the lack of support for modern computers in aerodynamic design of high speed jet aircraft and missiles as well as for research in computational fluid dynamics. Towards the end of his five year stay at RAE, he became dissatisfied due to lack of support for applied mathematics and computational fluid dynamics from the British government and other national agencies. However, even as Director of RAE, he was actively involved in research and development of aerodynamics of the slender delta wing for aircraft, spacecraft, and performance of high speed jet aircraft and missiles.

In 1964, Sir James was selected for the position of Royal Society Research Professor at the Imperial College of Science and Technology at London. In a new and conducive academic atmosphere at Imperial College, he had the usual responsibility of teaching, research and guiding many promising young researchers and other visiting research scholars. As his career progressed he was noted for changing his research from one area to another. At Imperial College, he delivered three courses of lectures on Nonlinear Waves, Geophysical Fluid Dynamics, and Blood Flow in Arteries during 1965-1967. In addition to his many outstanding papers and presentations on these topics, he got involved in the development of strong research in mathematical biofluiddynamics. He was actively involved in understanding of the flow of blood in mammalian cardiovascular systems, of air in the human airways, and of the flying of insects and birds, and developing the standard model of fish swimming using body undulations. His small-amplitude, slender- (or elongated-) body theory describes how thurst is generated from the reactive forces experienced by an undulating body as it exerts sideways acceleration to fluid which is moving backwards relative to the fish at almost steady swimming speed. Lighthill took the opportunity to expand his interdisciplinary collaborative research efforts among engineering scientists, physical scientists, life scientist and medical doctors. His leadership role led to the establishment of a Physiological Flow Studies Unit at Imperial in 1966 under the directorship of Dr. Colin G. Caro. He was delighted to see that this Unit celebrated 25 years of successful work under Professor Caro's leadership in May 1991. From 1966, his work involved a close interdisciplinary collaboration on internal biofluiddynamics by doctors, engineers, physiologists, applied mathematicians within the unit and in closely related departments at Imperial College. However, he again became very unhappy with the level of support given to applied mathematics from government and other external sources for interdisciplinary research. Under his unique leadership, a new *Institute of Mathematics and Its Applications* was established in the United Kingdom in 1965, and he became the first President of this newly created Institute. During 1963-1965, he was awarded Royal Society Medal, Gold Medal from the Royal Aeronautical Society, and Timoshenko Medal from the American Society of Mechanical Engineers for his many outstanding research contributions. He served as Physical Secretary and Vice President of the Royal Society of London during 1965-1969.

In 1967, Lighthill participated in the IAU Symposium with a lecture on "Predictions on the velocity field coming from acoustic noise and a generalized turbulence in a layer overlaying a convectively unstable atmospheric region." He presented the general linear non-dissipative theory of propagation of anisotropic waves through horizontally stratified medium. Included were fundamental wave motions under the influence of compressibility, gravity and a magnetic field and wave generation in and above the convection zone where the turbulence is relatively homogeneous. There are regions above the convection zone where intermittent turbulence is present. Both gravity waves (also known as *internal waves*) and Alfven's magnetohydrodynamic waves can be generated in this region. This is followed by the attenuation mechanisms that exist for linear and nonlinear dissipation of wave energy into heat. When magnetohydrodynamic waves are generated, either directly or by the transformation of sound waves or internal waves into them, magnetic pressures and gas pressures would be comparable so that there exists region for onset of nonlinear effects. In addition to waves generated by linear theory, random nonlinear dissipation mechanisms can generate fast hydromagnetic waves that would in turn produce shock waves. This lecture was published in the *IAU Symposium* No. **28** (1967) 429-453 (*Aerodynamic Phenomena in Stellar Atmosphere*), Academic Press.

From Imperial College, Lighthill returned to Cambridge in 1969 to succeed Paul Dirac (1902-1984), 1932 Nobel Prize winner in Physics as the Lucasian Professor of Applied Mathematics. Paul Dirac was one of the founders of quantum theory and the author of many of its major subsequent developments. He discovered the relativistic equation for the electron which is universally known as the Dirac equation. He is ranked with Isaac Newton (1642-1727), Albert Einstein (1879-1955), James Maxwell (1831-1879), Max Planck (1858-1947), Ernest Rutherford (1871-1937) as one of the greatest theoretical physicists of all time. Dirac also first discovered the existence of a positron — an antielectron having the same mass and opposite charge as the electron which was experimentally confirmed by the 1936 Physics Nobel Prize Winner, Carl Anderson in 1932. The Lucasian Chair is probably the most prestigious Mathematics Chair in the United Kingdom as this Chair was formerly held by many renowned British scientists including Sir Isaac Newton, Sir George Airy (1801-1892), and by Sir G. G. Stokes (1819-1903). Remarkably, Sir G. G. Stokes taught over forty-five future Fellows of the Royal Society, most notably Clerk Maxwell, Lord Rayleigh and J. J. Thompson. It was under Stokes' tenure as the Lucasian Chair that Cambridge became the major center of British applied mathematics and physics, just as the Mathematical Tripos and Natural Science Tripos evolved from a liberal education into specialized training for a scientific research career. Stokes became a popular public figure in Victorian Great Britain since he was a prominent scientist and a loyal Christian. Stokes was President of the Royal Society and served as an editor of the *Philosophical Transactions of the Royal Society* for thirty years and, indeed, was a real statesman of applied mathematics and science. Lighthill was justifiably proud to tell people that his predecessor in the Lucasian Chair was Newton. It is not out of place to point out that Stephen Hawking succeeded Lighthill as the Lucasian Chair in 1979. Stephen Hawking is widely considered as one of the most brilliant scientists since Newton and Einstein. He is a Cambridge based theoretical physicist whose major contributions provided a greater and deeper understanding of the complex nature of the Universe. It is a delight to quote his famous question: "... where did the universe come from? How and why did it begin? Will it come to an end, and if so, how? These are questions that are interest to us all." Through his many publications of bestselling popular science books, he has become the modern world's renowned science story writer in ordinary language.

Within the next two years of his stay in Cambridge, Queen Elizabeth conferred him a Knighthood as Sir James Lighthill in 1971. Among his many other awards and honors were the Bakerian Lecture on Sound generated aerodynamically in 1961, the 48th Wilbur Wright Memorial Lecture on Mathematics and Aeronautics in 1960, the Wright Brothers Lecture on Jet Noise in the United States in 1963, the Symons Memorial Lecture on Unsteady wind-driven ocean currents in 1969, the inaugural Frederic Constable Lecture in 1980, American Institute of Aeronautics and Astronautics (AIAA) Aeroacoustics Award in 1976. During his stay in Cambridge from 1969 to 1979, Sir James vigorously continued his teaching and research on acoustics, more and more wave propagation, geophysical fluid dynamics, biofluid mechanics, ocean and atmospheric dynamics with special reference to prediction of monsoons and tropical cyclones. He served on the British Post Office Board for a period of two years (1972-1974) in order to help promote the long distance telephone direct dialing system, commercial use of television and communications satellites in the United Kingdom.

In addition, Lighthill established two new collaborative research groups including the aquatic animal locomotion group with Sir James Gray of Zoology Department of the University of Cambridge and the animal flight group with T. Weis-Fogh of the Department of Zoology in Cambridge. Lighthill also recognized the brilliant discovery of a new mechanism of lift generation involving formation of trailing vortices at the wing tips, and including the case of a hovering insect like *Encarsia formosa* (a small parasite used in the biological control of greenhouse aphide) moving those tips in circular paths. This work of Weis-Fogh represents a fundamental contribution to knowledge of animal flight, in general and hovering flight, in particular. He described the role of the ‘clap’ and ‘fling’ motions in *Encarsia formosa*: that is, the clappings of the wings behind the back of the insect and their subsequent ‘fling open’. Indeed, this led to a fundamentally new biofluiddynamic mechanisms by which they operate to generate instantaneously an exceptionally large wing lift. No doubt, Lighthill was an outstanding promoter of interdisciplinary collaboration during his stay in Cambridge as James Gray, G. I. Taylor, T. Weis-Fogh, G. J. Hancock were involved in interdisciplinary research between applied mathematics and other diverse areas including biology, zoology, and biophysics. He himself made fundamental contributions to biofluid mechanics and flagellar hydrodynamics.

Based on his many invited presentations and inaugural addresses, Sir James wrote a large number of articles and texts of addresses that I find informative and interesting additions to his total scientific work. Obviously, the magnitude of the presentations demands some selection of the articles to be included in this book. However, any serious discussion of his all addresses would require more space and time and competence that I have. So I would like to focus on some of his addresses with a little more elaboration.

His 1961 Bakerian lecture to the Royal Society of London on Sound generated aerodynamically deals with Sir James’ original work on modern aerodynamics with new practical applications. Included are new developments in the frontiers between acoustics and aerodynamics, reduction of jet aircraft noise and improved knowledge of space-time correlations in turbulent flow that is used to throw new light on the noise radiated by turbulent boundary layers, as well as by jets at high Mach numbers. This lecture was published in the *Proceedings of the Royal Society of London*, volume **A267** (1962) 147-181.

Lighthill was selected to deliver the Fifth Ludwig Prandtl Memorial Lecture on "A Technique for Rendering Approximate Solutions to Physical Problems Uniformly Valid" on April 7, 1961. This Memorial Lecture commemorates Prandtl the great Physical Scientist of the twentieth century who made the revolutionary discovery of the boundary layer theory in 1904, the *annus mirabilis* that totally transformed the nineteenth century fluid mechanics. His discovery conclusively resolved the major controversy of the famous d'Alembert Paradox that an inviscid potential flow in three-dimensions around a rigid body moving at a uniform velocity exerts no resistive force on the body. Mathematically, $\mathbf{F} = -(d\mathbf{P}/dt) = 0$, where $\mathbf{P} = m\mathbf{v}$ is the total momentum of the fluid and $\mathbf{F}$ is the total external force transmitted to the fluid by the body. However, the behavior of the experimentally predicted flow is quite different from that of the potential flow. This fallacy lies not in the direct neglect of viscous forces, but rather in the assumption that there is *no* vorticity in the fluid outside the body. A body moving through a real fluid has behind it a wake containing vorticity. If the flow around a steadily moving body could be made quite close to a potential flow, the resistive force would become very small, but *not* zero.

Prandtl's discovery also resolved another erroneous prediction of the theory that fixed-wing aircraft would be unable to fly, because the air cannot exert any force at all — resistive or lifting force on a steadily moving body. Finally, the boundary layer theory explained how the nature of thin wakes from well designed wing shapes could allow wings to exert large forces (lift forces) normal to the flow, even resistive forces would continue to be small, but not zero, in the general spirit of de Alembert's Paradox. Therefore, major controversies of theory and experiment of the nineteenth century fluid mechanics came to an end. At the same time, Prandtl first introduced a totally new method of treatment of one kind of singular perturbation theory which provided considerable interest far outside his own field of fluid dynamics. Prandtl first developed a new correct approximate expansion method which involved the so-called boundary layer flow near a solid surface of an inviscid fluid flow away from the surface. This matching technique is also called the Prandtl *boundary-layer method* or the singular perturbation expansion method. This method deals with an approximate solution which is valid in a limited region. Another quite different approximate solution is obtained so that it becomes valid in another region that overlaps the first region. After adjusting arbitrary constants, two solutions are matched as closely as possible in the overlapping region.

On the other hand, there is another kind of a new approximate solution valid throughout the region of interest. In this case, an approximate solution valid only outside a certain region is sought, then we seek a method for rendering it uniformly valid. So, there are two different kinds of approximation methods possible for the so-called *singular perturbation problems.* In his lecture, Lighthill gave a modern mathematical treatment of singular perturbation theory which was successfully applied to resolve a long-standing enigma in fluid mechanics of the twentieth century. More generally, the singular perturbation theory is also successfully applied to nonlinear ordinary or partial differential equations with independent contributions of Rayleigh (1910) and Taylor (1910) to the internal structure of shock waves. From a mathematical point of view, Prandtl's original solution, besides representing an extremely early example of a singular perturbation, was successfully applied to fairly general nonlinear field equations. A similar revolution in knowledge of nonlinear effects in the generation and propagation of waves in fluids including sound waves and water waves has been originated from the mathematical analysis of shock waves. Indeed, all these represented outstanding examples of singular perturbation theory that was applied to nonlinear field equations involving partial differential equations. From a physical point of view, Prandtl's new ideas and insights led to the introduction of 'streamlined' shapes deviating from potential flow fields would experience very low resistive forces. Furthermore, Prandtl made the first qualitative as well as quantitative understanding of drag and lift forces on fixed-wing aircraft based on sound physical principles. He progressively revolutionized the major understanding of aerodynamic lift and drag forces and of fluid flows at high Mach numbers. So, the Prandtl's discovery has served as the fundamental basis for all crucially important subsequent developments of modern fluid mechanics. His superb work on fluid mechanics was subsequently published in an extended 1952 English-language edition as *Essentials of Fluid Dynamics.* In order to recognize Prandtl's brilliant contributions to fluid mechanics, Lighthill's comment is worth quoting: "... Indeed, his revolutionary discovery of the boundary layer in 1904 had the same transforming effect on fluid mechanics as Einstein's 1905 discoveries had on other parts of physics."

His next major contribution was the Wright Brothers Memorial Lecture on Jet Noise at the 31st Annual Meeting of the Institute of Aerospace Sciences in January 21, 1963. This lecture deals with the theoretical results with experimental evidence of how the shearing motions in a turbulent jet allow to shed some of their energy as sound radiation that is then propagated away through the atmosphere. Included are both subsonic and supersonic jet noise theories with the reduction jet aircraft noise. This is followed by a recent development, refinement and extension of Lighthill's theory to other areas of fluid mechanics, and to astrophysics where acoustic radiation in stars and in cosmic gas clouds is important. This lecture was subsequently published in *AIAA Journal*, **1** (1963) 1507-1517.

It is noteworthy to mention Sir James' contribution of a review article on Turbulence to the edited volume, *Osborne Reynolds and Engineering Science Today* published by Manchester University Press in 1970. Edited by D. M. McDowell and J. D Jackson, this volume represents a Proceedings of the major symposium held at the University of Manchester to celebrate the 1868 centenary appointment of Osborne Reynolds as the first new University Chair of Engineering. The University of Manchester established a great tradition of engineering science, applied mathematics and fluid mechanics by appointing Osborne Reynolds to this new University Chair of Engineering in 1868, and Sir Horace Lamb to Chair of Applied Mathematics in 1885. Both Reynolds and Lamb made many revolutionary contributions to engineering science, applied mathematics and fluid mechanics. Many original paper of Reynolds dealt with fundamental role of the Reynolds number in instability and transition, and of the Reynolds stresses in fully developed turbulent flows and of the Reynolds analogy in turbulent heat transfer. Among many others, the sixth edition of his book *Hydrodynamics* was completed by Sir Lamb at his home in Selwyn Gardens, Cambridge after retiring from Manchester and it was then published by Cambridge University Press in 1932.

In addition to Reynold's fundamental classical work, Lighthill described some major contributions of Rayleigh, Taylor, Prandtl, von Kármán, Landau and others with special emphasis on instability and transition to turbulence. Taylor first made his pioneering theoretical and experimental studies of stability of viscous fluid flow between two coaxial cylinders rotating at different velocities (see Debnath et al. (2001)). His beautiful experimental work revealed the existence of instability that led to a steady secondary flow in the form of a series of toroidal vorticities (Taylor vortices). These vortices are rotationally symmetric about the axis of the cylinders and spaced periodically with alternating spin along the axis. The basic cellular pattern of Taylor vortices depends on the primary flow before instability being two-dimensional. On the other hand, the stability problem of parallel viscous fluid flows was much more difficult partly because of the serious analytical difficulty of solving the Orr-Sommerfeld equation which governs the problem. In the mean time, Prandtl made exceedingly successful applications of the knowledge about turbulence to problems of external aerodynamics. He also explained the existence of a critical Reynolds number for fluid flow around bluff bodies, above which transition to turbulence in the boundary layers delays separation and significantly reduces drag force. The von Kármán school in Pasadena, California was also very active in finding information on mean velocity profiles in turbulent flow. In the mid thirties, Taylor initiated first to generalize the classical idea of the Reynolds stress which is the mean product, $\overline{u'(\mathbf{x})u'(\mathbf{x}+\mathbf{r})}$ of two disturbance velocities at points separated by a distance vector $\mathbf{r}$. Taylor's most fundamental idea of diffusion by continuous movements was to show how to describe diffusion process in terms of the statistical properties of fluid velocity. To describe turbulent motion of fluid, Taylor represented fluid velocities in the Eulerian manner as continuous random functions of time and position so that it simplifies greatly the expressions for the instantaneous spatial derivatives of velocity which occur in the equation of motion and in the vorticity and the rate of dissipation. This Taylor's new approach was almost subsequently and greatly extended by many others in late thirties and early forties. Taylor also published a series of original papers entitled 'Statistical Theory of Turbulence'. Indeed, all subsequent developments of turbulence have been based on Taylor's original approach and ideas.

The actual difference between 'restrained' and 'abrupt' transition has been recognized and then explained by the great Russian physicist Landau in 1944. Based on his famous nonlinear model equation, he explained that an amplitude A of small disturbances changes *exponentially* like $\exp(\gamma t)$ where

$$\gamma < 0 \quad \text{when} \quad R < R_c \quad \text{and} \quad \gamma > 0 \quad \text{when} \quad R > R_c, \tag{1.1}$$

the Reynolds number R reaches a critical value R_c which is relatively high. The square of the amplitude A proportional to the energy associated with disturbances to the mean flow satisfies the nonlinear Landau equation in the form

$$\frac{dA^2}{dt} = 2\gamma A^2 - \alpha A^4, \tag{1.2}$$

where γ is the rate of exponential increase of amplitude given by the small-amplitude theory and α is constant. For very small A, the right hand side of (1.2) is dominated by the term, $2\gamma A^2$ so that it is totally consistent with the behavior of (1.1) for infinitesimal disturbances. However, the term $(-\alpha A^4)$ in (1.2) provides how the growth of amplitude A may be either become restrained ($\alpha > 0$) or abrupt ($\alpha < 0$).

Substituting $B = (1/A^2) - (\alpha/2\gamma)$ in (1.2) gives the simple form

$$\frac{dB}{dt} + 2\gamma B = 0. \tag{1.3}$$

This admits the general solution

$$B(t) = a \exp(-2\gamma t), \tag{1.4}$$

where a is an arbitrary constant. Consequently, the general solution of (1.2) reduces to the form

$$A^2 = \left[\left(\frac{\alpha}{2\gamma}\right) + a\, e^{-2\gamma t}\right]^{-1}. \tag{1.5}$$

For subcritical Reynolds numbers $R < R_c$, $(\gamma < 0)$ and the energy of disturbances (1.5) tends to zero as $t \to \infty$. For supercritical Reynolds numbers $R > R_c$, $(\gamma > 0)$ and then all solutions (1.5) tend to a finite value of disturbance $A^2 = (2\gamma/\alpha)$. Any very small disturbance grow exponentially, but does not continue indefinitely. In fact, after a certain time, a definite level of disturbance is attained such as Taylor vortices between rotating cylinders for example.

Thus, a restrained transition $(\alpha > 0)$ is possible where very small disturbances with $(\gamma > 0,\ R < R_c)$ begin by growing exponentially, but, after a certain time, a definite level of disturbance is maintained. On the other hand, an abrupt transition becomes possible, where disturbances with $(\gamma > 0,\ R < R_c)$ decays to zero.

The Kármán vortex street in a wake, similar vortex rows in jets, Taylor vortices in flow between rotating cylinders, and various types of cells formed in a layer of fluid heated from below are interesting examples of restrained transition, while transition in pipes, channels and boundary layers is normally abrupt in the sense that an almost fully developed turbulent motion occurs.

Modern theories of turbulence began with the pioneering work on the universal equilibrium theory of the great Russian mathematical and physical scientist Kolmogorov in 1941. He developed a fairly general theory based on the Navier-Stokes equations and the continuity equation, and on the assumption of the existence of the main energy containing 'large eddies' (large scale or small wavenumber) and self-similar 'small eddies' (small scale or large wavenumber) which are primarily responsible for the viscous dissipation of energy. The Fourier spectral analysis of the turbulent energy spectrum $E(k,t)$ representing the contributions to the kinetic energy with respect to the wavenumber (or scale) k is the central problem of the dynamics of turbulence. It is important to point out that the nonlinear (convective) term in the Navier-Stokes equations is the main source of energy transfer from large eddies to small eddies. Denoting the nonlinear transfer term at a wavenumber k by $T(k,t)$, the energy spectrum $E(k,t)$ in the three dimensional isotropic turbulence satisfies the evolution equation (see Debnath (1998))

$$\frac{\partial E}{\partial t} = T(k,t) - 2\nu k^2 E(k), \tag{1.6}$$

where terms of this equation represent contributions of the inertial, nonlinear and viscous term of the Navier-Stokes equation. It follows from the continuity equation that the pressure term in the Navier-Stokes equations has no contributions to (1.6). This means that the net effect of the pressure field is to conserve the total energy in the wavenumber space. Only the nonlinear term in the Navier-Stokes equation is responsible for energy transfer from larger to smaller eddies — a mechanism by which larger eddies decay. The second term on the right hand side of (1.6) represents the dissipation of energy by molecular viscosity. Thus, the action of viscosity leads to a decrease in the kinetic energy of disturbance with the wavenumber which is proportional to the intensity of the disturbance multiplied by $2\nu k^2$.

It follows from the conservation of energy by the nonlinear term in the Navier-Stokes equations that

$$\int_0^\infty T(k,t)dk = 0, \tag{1.7}$$

so that the evolution equation (1.6) gives

$$\frac{d}{dt}\left(\frac{1}{2}\,\overline{u_i^2}\right) = \frac{\partial}{\partial t}\int_0^\infty E(k,t)dk = -\varepsilon(t), \tag{1.8}$$

where $\varepsilon(t)$ represents the rate of energy dissipation and is given by

$$\varepsilon(t) = 2\nu \int_0^\infty k^2 E(k,t)dk. \tag{1.9}$$

This shows that small eddies (or large wavenumber components) are dissipated more rapidly by viscosity than large eddies (or small wavenumber components). The basic assumption of the Kolmogorov theory is that at a very high Reynolds number, the turbulent flow at the very small scales (large wavenumbers) is approximately similar to a state of statistical equilibrium. Further, the motion of the small eddies is assumed to be statistically independent of those of in the energy-containing range. The energy containing scales of the motion may be anisotropic and inhomogeneous, but these features are completely lost in the cascade of energy transfers so that at much smaller scales the motion is locally homogeneous and isotropic. Hence the statistical properties of the turbulent motion in the equilibrium range must be completely determined by the two physical parameters ε and ν only that are relevant to the dynamics of this part of the spectrum only. Using the dimensional analysis, the fundamental length characterizing the energy-dissipating eddies must be $\eta = (\nu^3/\varepsilon)^{\frac{1}{4}}$. This length scale is known as the *Kolmogorov dissipation length* . The same dimensional analysis also reveals that the energy spectrum $E(k)$ depending on k, ε and η takes the form

$$E(k) = C_k\, \varepsilon^{2/3} k^{-5/3} F(k\eta), \tag{1.10}$$

where F is an universal dimensionless function.

According to Kolmogorov's hypothesis, for sufficiently large Reynolds number, there exists a significant range of wavenumbers with $k_\ell \ll k \ll k_d$ where $k_\ell = \ell^{-1}$ (ℓ is the largest scale) and $k_d = \eta^{-1}$ (η is the smallest scale). So, in this range, both energy content and energy dissipation are negligible and the spectral energy $\varepsilon(k) = \varepsilon$ is independent of wavenumber k. The molecular viscosity ν then becomes insignificant, $F(k\eta)$ is asymptotically equal to unity. Consequently, the Kolmogorov energy spectrum in the inertial range reduces to the form

$$E(k) = C_k\, \varepsilon^{2/3} k^{-5/3}. \tag{1.11}$$

This is known as the *Kolmogorov-Oboukhov energy spectrum* for isotropic and homogeneous turbulence, and has received strong experimental support by Grant, Stewart and Moilliet (1962) with a value of $C_k \approx 1.44 \pm 0.06$. Subsequently, Metais and Lesieur (1992) proposed the structure-function model of turbulence with the spectral eddy viscosity based upon a kinetic energy spectrum in space. Their analysis gives the best agreement with the Kolmogorov $k^{-5/3}$ spectrum law and the Kolmogorov constant $C_k \approx 1.40$.

Soon after Kolmogorov's brilliant discovery, considerable progress was made on a detailed study of different physical mechanisms of energy transfer of turbulence. In particular, Onsager (1945) and Heisenberg (1948) elucidated further the viscous dissipation mechanisms of small eddies in the cascade of random energy transfer in turbulence. Of these physical transfer mechanisms, Heisenberg's eddy viscosity transfer was found to be more satisfactory at that time. Based on the assumption that the role of small eddies in the nonlinear transfer process is very much similar to that of molecules in viscous dissipation mechanisms, Heisenberg suggested that small eddies act as an effective viscosity produced by the motions of these small eddies and the mean square vorticity associated with the large eddies. Using this assumption, Heisenberg formulated the energy balance evolution equation in the form

$$\frac{\partial}{\partial t}\int_0^k E(k,t)dk = -2\left(\nu + \frac{\eta_k}{\rho}\right)\int_0^k k^2 E(k,t)dk, \tag{1.12}$$

where η_k is the eddy viscosity defined by Heisenberg in the form

$$\eta_k = (\rho\kappa)\int_k^\infty \left[\frac{E(k,t)}{k^3}\right]^{\frac{1}{2}} dk, \tag{1.13}$$

where κ is a numerical constant.

However, Landau recognized some major difficulties with the Kolmogorov universal equilibrium theory of turbulence, and pointed out that statistics of small scales depend on those of large scales, and hence, it cannot be fully universal in the sense of the Kolmogorov assumptions. His other concern was that intermittency of dissipation increases with Reynolds number. In respond to Landau's concerns, both Kolmogorov (1962) and Oboukhov (1962) suggested some modifications of their original theories which led them to derive the so called *log-normal spectral* law in the form

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3} \ln(k/k_I)^\beta, \tag{1.14}$$

where only a fraction of β of energy is transferred from one scale to another and k_I is the wavenumber at which energy is injected. This is a slight modification of the $(-5/3)$ law in the sense that experimental measurements can hardly detect it. In spite of the above and other criticisms, the Kolmogorov $k^{-5/3}$ energy spectrum law received strong experimental support over a wide range of Reynolds number.

In spite of convincing and accurate arguments of Kolmogorov, the mathematical expression of Onsager's cascade hypothesis has subsequently been modified by Pao (1965) who began with the equilibrium equation

$$\frac{dS(k)}{dk} = -2\nu k^2 E(k), \tag{1.15}$$

where $S(k)$ is the turbulent energy flux from wavenumbers less than k to wavenumbers greater than k, and $2\nu k^2 E(k)dk$ represents the rate of viscous energy dissipation in wavenumbers between k and $k + dk$. Pao's argument was that $S(k)/E(k)$ must be a function of ε and k only because energy transfer from one wavenumber to another is essentially due to inertial effect in which viscosity has no explicit role. Based on dimensional analysis with $S(k)$ representing energy transfer per unit time and $E(k)$ is an energy per unit wavenumber, Pao formulated that

$$S(k)/E(k) = C^{-1}\varepsilon^{\frac{1}{3}}k^{\frac{5}{3}}, \tag{1.16}$$

where the constant must be reciprocal of that in (1.11) so that $S \to \varepsilon$ for small (ηk). Consequently, equations (1.15) and (1.16) can be shown to imply the explicit form of the spectrum for the entire Kolmogorov equilibrium range of wavenumbers in the form

$$E(k) = C\varepsilon^{2/3}k^{-5/3}\exp\left[-\frac{2}{3}\,C(\eta k)^{4/3}\right]. \tag{1.17}$$

This result also received remarkably a good agreement with several experimental observations. In spite of remarkable theoretical and experimental success of the Kolmogorov universal equilibrium theory of turbulence, Lighthill expressed a major concern about isotropic nature of small eddies. Thus, the Kolmogorov theory deals with only isotropic turbulence which is just one of more kinds of turbulence, by no means *generic*. In shear flow, there is a striking contrast between the isotropic small eddies and the big eddies that are highly elongated in the direction of the stream. Indeed, the main energy containing eddies are definitely *anisotropic* in shear flows. So, there are some questions about the validity of the Kolmogorov theory. Despite these criticisms, it must be recognized that the Kolmogorov theory is, after all, an approximate theory based on suitable assumptions and approximations, and has served as the fundamental basis for all subsequent developments in the modern theories of turbulence.

Recognizing all remarkable ideas and methods of Reynolds' and others, Lighthill described all current major developments of chaos and turbulence made by many eminent scientists including G. K. Batchelor, S. Chandrasekhar, W. Heisenberg, Th. von Kármán, A. N. Kolmogorov, L. D. Landau, C. C. Lin, L. Onsager, L. Prandtl, Lord Rayleigh, G. B. Schubauer, G. I. Taylor, and W. Tollmien. He emphasized the major work of Reynolds on turbulence with all subsequent advances that have been made by methods involving extensions of the original ideas in Reynolds' own research and his wide knowledge and expertise in the area of statistical mechanics. Finally, he concluded this article suggesting a number of possible new directions of advance with a statement that turbulence is one of the hardest areas of fluid mechanics and so, future progress will always be slow and "... Only very occasionally are there really new insights. I believe, furthermore, that none which I have described today can be regarded as comparable in magnitude with initial progress made by Reynolds himself. It is satisfying, furthermore, at these centenary celebrations to recognize how many of the later advances have been made by methods involving extensions of the original ideas in Reynolds's own work."

Lighthill delivered his Presidential Address at the Centenary Meeting of the Mathematical Association of Great Britain in London, April 14, 1971. This meeting has been a great event of major significance in an international context. He described many effective activities of the Association during the last 100 years including many continuous improvements in British mathematical education during that period. His lecture also emphasized that the Association exercised a continuing influence on a vast range of activities in mathematics education, while also being able to participate in developing the Art of Teaching Mathematics at all levels. During the last 100 years, the British mathematical scientists provided a major leading role in a wide range of research in mathematics, statistics, physics and engineering science. For example, mathematical analysis was brilliantly developed by G. H. Hardy and J. E. Littlewood, geometry by H. F. Baker and William Hodge, number theory by L. J. Mordell and Harold Davenport, topology by Henry Whitehead and Max Newman, and mathematical philosophy by Alfred North Whitehead and Bertrand Russell. On the other hand, many revolutionary research have been carried out in Great Britain in mathematical physics by Paul Dirac and R. H. Fowler, in engineering science by G. I. Taylor and Sydney Goldstein and in statistics by R. A. Fisher and Frank Yates, and in earth sciences by Harold Jeffreys and Sydney Chapman. According to Sir James, the degree of integration of pure and applied mathematics in British universities has traditionally been very high by world standards. The Mathematics Association has continuously been engaged in improving the current trends in both primary and secondary education, and in expressing concern with how the mathematics being taught can be applied. The emphasis on the idea of building mathematical models representing one aspect of applied mathematics has been widespread all over the world as well as in Britain. One of his major motivations in this lecture was to illustrate diverse and newer applications of mathematics in physical and social sciences, economics, commerce and industry. Sir James also stressed the use of electronic computers in teaching and research that would provide new opportunities for curriculum development. Based on his lecture, he published an article on the Art of Teaching the Art of Applied Mathematics in *Mathematics Gazette* in volume **55** (1971) 471-492. He concluded this lecture by stating his comment as follows: "... to give careful thought and consideration and discussion to all matters concerned with this critical question of how to teach the art of applying mathematics."

His article on the Interaction between mathematics and society was published in the *Proceedings of the Third International Congress of Mathematics Education*, Universität Karlsruhe in 1977. Sir James published another article entitled Teaching how to make and use mathematical descriptions of engineering systems in the *Proceedings of the Second International Congress of Engineering Education* in 1978. Through his presentations and publications of articles on pedagogy, Sir James provided a vital leadership role in promoting not only the importance of teaching and learning at all levels, but also in enhancing the interaction between mathematics and society. His philosophy of mathematics education can be best described by citing his own quotation from his Presidential Address at the Second International Congress on Mathematical Education in Exeter in September, 1972.

"Let's go *beyond mere use* of the concrete example as an aid to understanding or of reference to utility as an aid to widening the circle of those in whom interest is aroused. There is a still more important prize to be won: a prize concerned with a *deeper* integration of mathematics into the total education of the individual."

"I want to suggest that educators may have most benefited their pupils when they have succeeded in giving them a feel for what is involved in the *process of applying mathematics*... Computers may be of great value in problem-solving, but apparently the human brain alone is able to tackle the subtler aspects of creating an effective correspondence between the mathematical world and the world of experiment and observation."

In July 1972, Sir James delivered the Fourth Annual Fairey Lecture on the propagation of sound through moving fluids at the Institute of Sound and Vibration Research in Southampton. Special attention has been given to the interaction of sound waves with turbulence and to further intensification of collaborative efforts between theory and experiment in the subject. This is followed by questions of the relative properties of upstream and downstream propagation of sound both in one-dimensional case and in the more fully three-dimensional cases characteristic of higher frequencies. He concluded his lecture with recent progress on work in the whole field of aero-engine compressor noise reduction as the compressor noise is a major part of the total external noise field of many modern aircraft. This lecture was published in *Journal of Sound and Vibration*, **24** (1972) 471-492.

Sir James Lighthill delivered a special lecture on Ocean Science at a conference held in Greenwich on September 12-14, 1973 to celebrate the hundredth anniversary of the Royal Naval College. In this lecture, he brilliantly described the major role of ocean science in the service of mankind and elaborated the interaction among the many disciplines of mathematics, science and engineering. His major focus includes (i) exploration for oil and gas, (ii) under-sea minerals, (iii) ocean's living resources including protein sources, fish and shell-fish, and other huge untapped resources, (iv) world's fish farming and fish industry, (v) ocean-borne trade and ship design technology, (vi) coastal protection against floods caused by violent winds and high tides, (vii) natural hazards and tsunami waves due to under-water earthquakes, (viii) sand and sediment movements, the effect on coastal erosion and on fisheries of dredging for sand and gravel, (ix) marine pollution and pollution control, world's clear air and world's clean water research and legislation, (x) discharges of oil in oceans and seas, and other industrial wastes, and dangers to marine life, (xi) air-ocean interaction and man's natural environment, (xii) more research on the atmosphere-ocean circulation, weather forecasting, flood prediction and other related topics, (xiii) biogeography dealing with the ranges of species and evolutionary processes, picture of the abundance of different species in different parts of the ocean, and importance of fishery science, (xiv) global geophysics and geology, speeds of ocean-floor spreading, high concentration of minerals in water and commercial exploitation of deep-ocean mineral resources, and (xv) the theory of continental drift and problems of waste disposal. In addition to all of the major topics directly associated with ocean science, he stimulated the participants of this centenary celebration for more advanced study and research on many challenging problems in interdisciplinary ocean science. In this context, his concluding statement is worth quoting: "With that visionary idea, which re-emphasizes my two main themes: ocean science in the service of mankind, and the endless interaction between all the disciplines that make up ocean science, I may perhaps conclude my presentation". Sir James subsequently published this presentation in the *Bulletin of the Institute of Mathematics and Applications* in February 1973 and also in the *Journal of Navigation*, **27** (1974) 91-110. Through publication of these articles in international journals, he presented this material before the multinational and multidisciplinary group of scientists for promotion of cooperative interdisciplinary research on ocean science and marine engineering.

Based on his major work published in the *Annual Reviews of Fluid Mechanics*, **9** (1969) 413-446, Sir James delivered an invited lecture on Aquatic Animal Locomotion at the Thirteenth International Congress of Theoretical and Applied Mechanics held in Moscow in 1973. In this lecture, he presented the interaction between an aqueous medium and the external surfaces of a totally immersed animal swimming through it. He also described importance of hydrodynamic theory relevant to aquatic animal locomotion and of the interdisciplinary research work between zoology and fluid mechanics. Lighthill first discussed his own reactive theory for swimming of animals with elongated bodies which was published in a series of papers based on a small-perturbation expansion about the stationary state. He also discussed the extension of the 'reactive theory' so that it no longer depends on a small-perturbation expansion. Some recent developments in two-dimensional theory that are relevant to lunate tails and their vortex wakes are also presented. Included are other major extensions of theories of motion of individual flagella at very low Reynolds number to the case of ciliary propulsion, where the movements of large number of attached cilia close to synchronism generate relative movement between an animal and the surrounding fluid medium. His lecture also included other topics such as hydrodynamics of ciliary propulsion, and large-amplitude elongated-body theory which deal with his work on motions at high Reynolds number and reactive force theories. Finally, he concluded his lecture by discussing vortex wakes that what aquatic animal locomotion leaves behind, and a diamond-shaped lattice pattern of fishes. Included is also a major contribution by von Kármán and Burgers (1934) which illustrates vortices cast off by the caudal-fin trailing edge as fish moves to the left, and also jet-like streamline pattern produced by those vortices.

Aerodynamic Aspects of Animal Flight was the title of Lighthill's lecture organized by the British Hydromechanics Research Association in 1974. He concentrated in this lecture on the aerodynamic forces, and resulting dynamic interactions, between the movements of animals flying in air and the surrounding air movements. Based on the basic aerodynamic requirements for sustained forward flight to produce sufficient lift to balance body weight and sufficient thrust to balance body drag, Lighthill presented the aerodynamics of sustained forward flight in birds, insects and bats with comments on the many other flight modes of these animals. These modes include manoeuvres, diving, soaring, take off, landing and hovering flight which are most fascinating aerodynamic aspects of animal flight. He concluded this lecture by suggesting a number of new directions of research in this field.

According to Lighthill, "there is immense scope for research in this new field, based on further exploration of the use of the "clap and fling" mechanism and other specialized unsteady aerodynamic effects by flying animals. It would be special interest to determine how widespread is the use of the mechanism by insects so small that they encounter low-Reynolds-number problems. A quite different question is whether the mechanism has been used at all by much larger animals, operating at substantial Reynolds number.... It would be a challenging problem to study by experiment and analysis what are special movements by which these 'thrips' or 'fringe-winged' insects are enabled, at such very low Reynolds numbers, to support their bodies in the air."

In his 1975 John von Neumann Memorial Lecture on Flagellar Hydrodynamics, Sir James presented a general overview of biofluiddynamic aspects of microorganisms with flagella and related organisms. This is followed by eukaryotic flagellar motions and bacterial flagellar motions with the structures of these two fundamentally different kinds of flagella. Special attention is given to application of mathematics to flagellar motions in particular. He concluded his lecture by adding a discussion on flow fields generated by flagella. This lecture was remarkably well attended and a considerable number of participants were not familiar with this new subject. So, many participants took the advantage of the unique opportunity to gain insights into the some of the underlying new ideas and results of the most fascinating and new branch of modern fluid mechanics. In addition to his own research work, however, the material embraced that of other fluid dynamists and zoologists both in the United Kingdom and in the United States. The treatment of the subject was to a large extend new and original contributions of the lecturer. His most remarkable work was subsequently published by *SIAM Review*, **18** (1976) 161-230.

In April 6, 1978, he was invited to deliver lecture on Acoustic Streaming at the spring meeting of the Institute of Acoustics in Cambridge, England. His lecture was published in the *Journal of Sound and Vibration*, **61** (1978) 391-418. Generally speaking, mean motions induced by sound waves are called *acoustic streaming* which is forced by the action of a Reynolds stress that can cause a net force per unit volume, F_j to act on the fluid. This force field can be written as $F_j = -\frac{\partial}{\partial x_i}\left(\overline{\rho u_i u_j}\right)$, where the repeated suffix i is to be summed from 1 to 3. This force field per unit volume is capable of generating a steady streaming motion. In fact, the equation for the

Eulerian mean motion $\overline{u_j}$ produced by this force is

$$\rho_0 \left(\bar{u_i} \, \frac{\partial \bar{u_j}}{\partial x_i} \right) = -\frac{\partial \bar{p}}{\partial x_j} + \mu \nabla^2 \bar{u_j} + \mu F_j. \tag{1.18}$$

This equation can be solved with the equation of continuity in the form

$$\rho_0 \frac{\partial \bar{u_j}}{\partial x_j} + \frac{\partial}{\partial x_j} \left(\frac{1}{c^2} \, I_j \right) = 0 \tag{1.19}$$

where the first term represents the Eulerian mean motion $\bar{u}_j$, and the second term corresponds to an acoustic energy flux I_j and c is the speed of sound.

This lecture also covered significance of several attenuation mechanisms and the acoustic streaming associated with the attenuation of sound waves. Sir James also elaborated by stating that all really significant streaming motions are studies of the Stuart (1963, 1966) streaming that is calculated from the full equation (1.18). In his pioneering work, Stuart discussed the nature of acoustic streaming based on the Reynolds number R_s on streaming velocity which has many modern ramifications of the classical work of the Rayleigh law of streaming. This is followed by a comprehensive treatment of patterns of turbulent acoustic streaming produced by ultrasonic sources and turbulent jets generated by attenuation of the energy flow in a general acoustic beam. He concluded his lecture with a review of modern developments in the other main type of acoustic streaming associated with the major interactions between sound waves and solid boundaries. These are still dominated by the Rayleigh law of streaming with the so called the *slip velocity* $\bar{u_s} = -\frac{3}{4} \, \omega^{-1} U(x) U'(x)$, where $U(x)$ is involved in any oscillating tangential relative velocity, $U(x) \exp{(i\omega t)}$ between a fluid and a solid boundary, whether caused by acoustic oscillations in the fluid or by vibrations of the solid. This leads to frictional dissipation within the Stokes boundary layer. The tangential component of the fluid velocity relative to the solid wall is given by

$$u = U(x) \exp{(i\omega t)} \left[1 - \exp \left\{ -z \left(\frac{i\omega}{\nu} \right)^{\frac{1}{2}} \right\} \right] \tag{1.20}$$

where $\nu = (\mu/\rho_0)$ is diffusivity of momentum. All these strongly indicate that Rayleigh's classical work has served as the fundamental basis for all modern developments in acoustic streaming.

In 1979, Sir James left Cambridge to serve in the position of Provost of the University College London (UCL). Many people were pleased with his appointment and expressed their feelings by saying that UCL got a Provost who has academic clout, visibility, and tremendous influence in government, industry and society. Due to his enormous wisdom and communication skills, and strength of character, he remained in that administrative position until his formal retirement in 1989. During this period, he was heavily involved in new developments of the College, in promotion of biological and biotechnological research, fund-raising activities, and in significant improvement of women's appointments in senior academic and administrative positions. Even though he was very busy with administrative duties and responsibilities, Lighthill still maintained his scientific research in areas such as extraction of wave energy, dynamical systems, and biomechanics of the human auditory system. In collaboration with a group of scientists in the Institute of Laryngology and Otology at the University College London, Lighthill continued his research on mammalian hearing biomechanics during his stay at UCL. From my personal conversation with him and his wife, Lady Nancy in October 1995, I was delighted to know that Sir James made many significant contributions to the University College London in many different ways.

While he was Provost, he was very active in the presentation of special lectures and in research and national educational affairs. His professional activities included services as President of the International Union of Theoretical and Applied Mechanics (IUTAM), as Member of the Advisory Board for the Research Council, 1980-1986, as Member of Natural Environment Research Council and Chairman of its Oceanography and Fisheries Research Committee, 1965-1970, as Member of Geddes Committee of Inquiry into Shipbuilding Industry, President of the International Commission on Mathematical Instruction (ICMI), 1971-1975, and as Member of the Advisory Council on Research and Development, 1978-1981. Fortunately, the United Nations has declared the period from 1990-1999 as the International Decade for Natural Disaster Reduction (IDNDR) and has adopted tropical cyclones as the major atmospheric hazard. After the International Council of Scientific Unions (ICSU) has strongly encouraged member Unions to support IDNDR, the IUTAM and the International Union of Geodesy and Geophysics (IUGG) have agreed to make a joint plan for a decade long scientific program related to tropical cyclone disasters. Sir James Lighthill provided a leadership role to develop a plan for a better tropical cyclone forecasting and warning system that minimizes the loss of human lives and other socio-economic losses associated with tropical cyclones at the IUTAM/ICSU/IUGG workshop in Vienna in August 1990. As Sir James Lighthill noted in his broad introduction at the ICSU and the World Meteorology Organizations (WMO) the first international symposium in Beijing

China in 1992, scientists from related fluid dynamics disciplines have an important role to play in the research challenge of the International Decade, especially by helping to improve the accuracy of techniques for detailed forecasting of Natural Disasters so that population under threat may learn to rely on the predictions, and consequently, to follow the emergency measures recommended for their protection.

As an active President of ICMI which was then a sub-commission of the International Mathematical Union (IMU), Lighthill has had a great influence on mathematics education and science policy at the international level. The emphasis on new thinking and pedagogical development became his top priority as President of ICMI. In order to promote mathematics teaching at all levels and to upgrade mathematics education all over the world, Lighthill helped organize a number of symposia that include 'New topics in applicable mathematics in Secondary Schools' in Luxembourg in 1973, and 'Mathematics and Language' in Kenya in 1974. He will never be forgotten for his impact on mathematics and its applications, but equally, he made major lasting contributions in raising public awareness of mathematics education, and through conferences and symposia, bringing mathematics education issues to the fore.

In his brilliant lecture on "The recently recognized failure of predictability in Newtonian dynamics" at the Discussion Meeting on Predictability in Science and Society, in 1986, Lighthill, speaking as President of the International Union of Theoretical and Applied Mechanics (IUTAM) presented the first systematic and persuasive arguments in support of the complete predictability of systems governed by the equations of Newtonian dynamics. He also stated that Sir Isaac Newton's discovery of fundamental mathematical and physical laws were published in his first book of *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) which is considered one of the greatest single contribution ever published in the history of physical sciences. This celebrated volume, usually called *Principia* or *Principia Mathematica* was completed over three hundred years ago and communicated to the Royal Society in the Spring of 1686 and then published in 1687. In it Newton not only put forward a new theory of how bodies move in space and time, but also developed the complicated mathematics needed to analyze these motions. In addition, he also formulated the laws of motion and a law of universal gravitation according to which each body in the universe was attracted toward every other body by a force that was stronger the more massive the bodies and the closer they were to each other. It was exactly the same force that caused objects to fall to the ground. According to his law, gravity causes the Moon to move in an elliptic orbit around the Earth and causes the Earth and the planets to follow elliptical paths around the Sun. It was the first Newton's book to contain a unified system of scientific principles explaining what happens on Earth and in the Universe. Sir James' lecture was well attended by a group of learned audience, but the subject was highly controversial with which a large number of participants were not completely familiar and so many took the advantage of this opportunity to gain insights into the underlying ideas of the most fascinating and fundamental branch of mathematics and physical science. This lecture was published in *Proc. Roy. Soc. London*, **A407** (1986) 35-50.

As a firm believer of Newtonian mechanics, Sir James' statement of public apology is an enlightenment to read:

"Here I have to pause, and to speak once again on behalf of the broad global fraternity of practitioners of mechanics. We are all deeply conscious today that the enthusiasm of our forebears for the marvelous achievements of Newtonian mechanics led them to make generalizations in this area of predictability which, indeed, we may have generally tended to believe before 1960, but which we now recognize were false. We collectively wish to apologize for having misled the general educated public by spreading ideas about the determinism of systems satisfying Newton's laws of motion that, after 1960, were to be proved incorrect. In this lecture, I am trying to make belated amends by explaining both the very different picture that we now discern, and the reasons for it having been uncovered so late."

In his memorable lecture of 1986, Sir James excited the audience by the following statement and quotations:

"Three different new directions of research initiated the discovery of chaotic behaviour in simple systems satisfying Newtons laws during the early part of the 1960s; although, admittedly, it was only 20 years later that the remarkably widespread occurrence of chaotic behaviour had become clear; to such an extent that, for example, at the 1984 Congress of the International Union of Theoretical and Applied Mechanics the specialized topic which was highlighted most strongly in the Congress programme was that of chaotic behaviour.

The first of these three new directions of research arose directly out of Poincaré's work on nonlinear perturbation theory, related to equations in Hamiltonian form for isolated systems of constant energy. Work initiated by the great Russian analyst Kolmogorov (1954), and pursued by his gifted colleague. Arnold (1963), had been aimed first of all towards filling in the gaps in Poincaré's proofs; and, indeed, all this work along with independent studies by Moser (1962) in Germany and America demonstrated that, even in the neighbourhood of resonances, perturbations did assume a distinctly regular pattern in the vast majority of cases. Nevertheless, gaps in that regular pattern did exist; namely, very small ranges of initial conditions for which the motion assumed a form described as chaotic or stochastic (that is, random). It was regarded as interesting that equations of motion that included absolutely no random element should possess solutions which behaved in such a random way. Initially, however, the matter was seen as something of a curiosity because the highly complex proofs used in the perturbation theory were valid only when the perturbations were restricted to being sufficiently small and, in this case, the ranges of initial conditions for which chaotic or stochastic motions occurred were very limited indeed.

The second new direction of research utilized the powerful computers that were by then available to compute solutions not just in cases when the perturbations were small, but also for much larger perturbations. This work, carried out, for example, by Greene (1979) in the U.S.A. and by Chirikov (1979) in the U.S.S.R., demonstrated that as the strength of the perturbations continued to grow there was a sharp increase in the range of initial conditions for which solutions behaved stochastically. Finally, at a certain level of the perturbation strength, the authors observed what they called a transition to global stochasticity, with all solutions behaving chaotically. I shall describe later what this amounts to in detail, but in the meantime will note the obvious fact that the new data made a big increase in the importance to be attached to chaotic or stochastic solutions.

These results on isolated systems of constant energy were of interest not only to astronomy but also to thermodynamics. The second law of thermodynamics envisages, of course, an increase in the randomness of motions experienced by an isolated system of molecules; that is, an increase in its entropy; but physicists had long supposed that large numbers of collisions between molecules were necessary to allow such randomization to occur. Now, with the wider understanding of how chaotic motions can develop, it is possible to see that collisions may not be essential. For example, the ionized gas between the Sun and the Earth with its extremely low density, producing an astronomically large mean free path between molecular collisions, may nevertheless in the presence of magnetic fields experience phenomena that are possible only with an increase in entropy. One of these, which spacecraft have observed, is the so-called 'bow shock wave' where the solar wind of charged particles emanating from the sun is abruptly slowed down where it first encounters the Earth's magnetosphere.

But that is a digression, which may on the other hand have provided a valuable reminder that Newtonian dynamics is applicable not only to systems of solid bodies but also to fluid systems, including ordinary gases and liquids. Ordinary gases and liquids, of course, are subject to the phenomenon called viscosity, which causes the mechanical energy in their shearing motions to be gradually dissipated into heat; precisely as a result of an entropy increase associated with normal molecular collisions on a submicroscopic scale. Yet even the damping of fluid motions by viscosity does not prevent perfectly regular fluid motions from becoming chaotic and this fact was first made precise over a century ago, in 1883, by Osborne Reynolds. He showed how the regular flow of fluid through a pipe suddenly becomes chaotic or turbulent when the force producing the motion becomes sufficiently large relative to the damping forces due to viscosity. He showed, furthermore, that this randomization has nothing to do with the random molecular movements occurring at submicroscopic scales; they, indeed, have a damping effect tending to reduce the trend towards turbulent motions; motions which themselves involve, rather, a chaotic pattern of fluid movement on a strictly macroscopic scale.

Thus, the specialists in dynamics of fluids, such as myself, have long been most fully conscious of the common tendency for regular or laminar motions of fluids to become chaotic or turbulent even though the motions in questions are subject to energy dissipation by the action of viscosity. On the other hand, fluids represent very complicated dynamical systems with an essentially unlimited number of degrees of freedom (each separate particle of fluid is separately free to be arbitrarily positioned relatively to all the other particles) and it had never been clear whether or not this was an essential pre-requisite for chaotic behaviour to develop.

Against that background it may be interesting to note that the third new direction of research which began in the early nineteen-sixties was concerned with some quite simple systems analogous to turbulence. These were dynamical systems with energy dissipation and just two or three degrees of freedom which, although forced in a perfectly regular way, responded in a completely chaotic way when a ratio of forcing effects: damping effects (a ratio similar to the Reynolds number introduced by Reynolds) was sufficiently large. Initially, they were devised by some noted experts in dynamics of fluids, including the great atmospheric scientist E. N. Lorenz, in order to mimic as closely as possible the development of turbulence in fluid systems. Lorenz (1963) introduced the term 'strange attractor' to describe the type of randomized motion which inexorably tends to develop.

More recently, a very general theory of these strange attractors has been produced for such systems, which unlike the isolated energy-preserving systems studied by Poincaré and others, are subjected both to forcing and to damping (see chapter 7 of Lichtenberg & Lieberman 1983). This theory suggests the steps by which regular motions develop into chaotic motions as some forcing:damping ratio changes. Often that takes place via an infinite sequence of so-called 'period-doubling bifurcations' which terminate, after just a finite change in that ratio, in a completely chaotic motion. Numerical computations have excellently confirmed these theories and demonstrated the strong tendency for systems of this type also to develop chaotic motions.

Now at this point I might easily feel tempted to enlarge upon all the immense variety of different types of chaotic systems and of transitions to chaos; and yet none of that would be relevant to the subject of this meeting. My objective in the time that remains to me is, rather, to focus upon certain properties which are common to all chaotic systems and which are relevant to the issue of predictability."

Lighthill used his lecture at this discussion meeting to criticize the view of the recently recognized failure of predictability in Newtonian dynamics. A thorough appreciation of Newton's great scientific achievement was shown by Sir James in his following concluding comments:

"I feel fully justified, therefore, in repeating that systems subject to the laws of Newtonian dynamics include a substantial proportion of systems that are chaotic; and that, for these latter systems, there is no predictability beyond a finite predictability horizon. We are able to come to this conclusion without ever having to mention quantum mechanics or Heisenberg's uncertainty principle. A fundamental uncertainty about the future is there, indeed, even on the supposedly solid basis of the good old laws of motion of Newton, which effectively *are* the laws of motion satisfied by all macroscopic systems. I have ventured to feel that this conclusion would be of interest to a Discussion Meeting on Predictability in Science and Society. For example, there might be some other discipline where practitioners could be inclined to blame failures of prediction on not having formulated the right differential equations *or* on not employing a big enough computer to solve them precisely *or* on not using accurate initial conditions; yet we in mechanics know that, in many cases where the equations governing a system are known exactly and are solved precisely, nevertheless however accurately the initial conditions may be observed prediction is *still* impossible beyond a certain predictability horizon."

In spite of controversial nature of the subject, Lighthill's explanations have been strongly supported by one of the great mathematical scientists of the twentieth century, A. N. Kolmogorov (1903-1987) who published an article on Newton and Contemporary Mathematical thought in the journal *Matematika v Shkole* 1982, No. 6, 58-64 and the same article was reprinted in the book of *Kolmogorov in Perspective* (2000). It is appropriate to quote Kolmogorov's thought:

"Newton not only made fundamental discoveries in the mathematical natural sciences that we need not go into here, since they are widely known, but he actually was the first to create a mathematics for the natural sciences in the sense of a system for the mathematical investigation of all mechanical, physical, and astronomical phenomena. Before Newton one could say only that this or that individual area of the natural sciences could be studied by mathematical methods. Of course, the ideas of Leibniz about the possibility of a mathematization of the whole of human knowledge were even more universal. But they bore no fruit precisely because of their absolute generality and lack of concreteness. On the subject of universal applicability and at the same time restrictedness see my article "Mathematics" in the *Great Soviet Encyclopedia*."

At the invitation of the American Society of Mechanical Engineers (ASME), Sir James delivered the 1989 Rayleigh Lecture on Biomechanics of Hearing Sensitivity at the ASME Winter Annual Meeting in Chicago, Illinois. Speaking as the immediate Past President of IUTAM, he presented an elaborate survey on several topics including cochlear anatomy, cochlear sensitivity, cochlear macromechanics, cochlear micromechanics, evoked otacoustic emissions, and micromechanical interpretation of hearing sensitivity. He concluded his lecture by stating: "Finally, I emphasize that this lecture on the biomechanics of hearing sensitivity has been concerned, not with how the brain in man and other mammals analyze the data coming to it along auditory nerve fibers, but with the initial capture of that data in the cochlea. The brain, needless to say, can produce all its miracles of interpretation only where it works on good initial data.... I believe that Lord Rayleigh would have been excited and heartened by both aspects of our improved appreciation of the biomechanics of hearing sensitivity."

Lord Rayleigh was undoubtedly a wide ranging physical scientist who mapped out all of the classical known fields of acoustics including the science of hearing in his 1896 monumental treatise '*The Theory of Sound*' in two volumes. These two volumes provided a comprehensive treatment of an enormous body of knowledge obtained from linear wave theory as applied to generation and propagation of sound waves. Indeed, linear theories of wave motion in general became well developed during the nineteenth century. These volumes, a delight to read, are loaded with perennially significant information and ideas including the general derivation of the energy propagation velocity and acoustic streaming. Both Rayleigh (1910) and Taylor (1910) made an independent discovery of the internal structure of shock waves. This discovery subsequently led to the modern study of nonlinear effects in the generation and propagation of waves in fluids including sound waves and water waves. On the other hand, this discovery also led to the singular perturbation approach to nonlinear field equations in fluid mechanics. Indeed, the revolution in fluid mechanics during the second half of the twentieth century originated from the applications of new singular perturbation theory to nonlinear field equations where progress would have been impossible without the use of such a radically new approach. According to Lighthill, Lord Rayleigh was undoubtedly one of the most brilliant and influential physical scientists of the nineteenth century.

Sir James Lighthill was the best choice for the Inaugural Sydney Goldstein Memorial Lecture at the University of Manchester in October 11, 1989. His first association with Goldstein was in 1943 in Cambridge, and then in 1945 when Lighthill joined Goldstein's fluid dynamics group at NPL to do research in aerodynamics and applied mathematics. After his appointment to the Beyer Chair of Applied Mathematics at the University of Manchester in 1945, Goldstein first recruited a strong group of applied mathematicians including Lighthill as Senior Lecturer in Mathematics. Among his many outstanding contributions in teaching, research and service to the University of Manchester, Sydney Goldstein in cooperation with Max Newman accepted the challenge of creating a dynamic and internationally renowned mathematics department at Manchester. At the same time, Goldstein spent his time and energy to build a strong group of applied mathematics and fluid mechanics in Manchester's great tradition established by Osborne Reynolds and Sir Horace Lamb. Many of his later contributions were made as an outstanding leader of a fine group of applied mathematicians including M. J. Lighthill, J. W. Craggs, F. G. Friedlander, C. R. Illingworth, D. S. Jones, R. E. Meyer, G. N. Ward and E. Wild.

In his memorial lecture on 'Some challenging new applications for basic mathematical methods in the mechanics of fluids that were originally pursued with aeronautical aims', Sir James described Sydney Goldstein's major research accomplishments in Manchester including his key role as editor of the big collective work of the two volumes of *Modern Developments in Fluid*

Dynamics: An account of Theory and Experiment Relating to Boundary Layers, Turbulent Motion and Wakes first published by the Oxford University Press in 1938. This two-volume paperbound is a unique classic and an indispensible introduction to laminar and turbulent fluid flows for graduate students and professionals in applied mathematics and engineering science. This basic work is not only offers an enormous essential fundamental information in fluid mechanics, but also provides a solid foundation for other specialized studies in the field. Sir Horace Lamb was originally selected to serve as the Managing Editor of this work, but when he relinquished this task, it was assigned to Sydney Goldstein. On the sudden death of Sir Horace in 1934, Goldstein became his successor in two capacities, one private and one public. He acquired the late Sir Horace's beautiful house in Selwyn Gardens and succeeded Lamb as the Managing Editor of *Modern Developments in Fluid Dynamics* that was originally planned by the Fluid Motion Panel of the Aeronautical Research Council. Most appropriately, this book was dedicated to the memory of Horace Lamb, and it does, indeed, represent an enduring memorial.

Sir James particularly mentioned Goldstein's famous work on the vortex theory of screw propellers and on laminar boundary layer flow near a position of separation which deals with the complete mathematical nature of solutions of the boundary layer equations near separation. In order to stimulate new theoretical and experimental research on turbulence, Goldstein had spent a year in Göttingen with the renowned fluid dynamist, Ludwig Prandtl to learn the enormous advances made by Prandtl's group in experimental work on fluid flow structures of engineering interest and their physical interpretation in terms of vorticity, and then offered a year long advanced course on the theory of turbulence in Manchester. Based on his own research, Goldstein published an original paper 'On the law of decay of homogeneous isotropic turbulence and theories of the equilibrium and similarly spectra' in *Proc. Camb. Phil. Soc.*, **47** (1950) 554-574. This is a highly original research work which provided some new implications of Kolmogorov's equilibrium theory for the decay of homogeneous and isotropic turbulence. The last two topics of Lighthill's lecture dealt with problems of biofluiddynamics including (i) vortex wakes of birds in flapping flight and (ii) some challenging problems in the field of animal locomotion with special reference to the nature of the velocity and pressure fields in near field as well as in far field of swimming fishes. He examplified the nature of the quadrupole for two cases of swimming clupeoid fishes: regular swimming with lateral oscillations of the caudal fin, and sudden movements by which a steadily swimming fish makes a turn. In both cases, Lighthill identified the expected nature of the quadrupole far fields in terms of a dipole acting at the tail or at the head, respectively, and an equal and opposite dipole acting located at the centroid. Sir James concluded the Goldstein Memorial Lecture with an interesting statement: "... in two biological sections of my lecture, I have indeed indicated 'some challenging new applications for basic mathematical methods in the mechanics of fluids, that were originally pursued with aeronautical aims' by great scientists including Sydney Goldstein." This lecture was subsequently published in the *Journal of Aeronautical Society*, **94** (1990) 41-52.

Among his many degrees, honors and awards, he received a Gold Medal from the Institute of Mathematics and Its Applications in 1982, and the Harvey Prize of the Israel Institute of Technology in 1981. In recognition of his notable research contributions to fluid mechanics and applied mathematics, he was elected to twelve learned societies including the US National Academy of Sciences, Russian Academy of Sciences, and the Indian National Science Academy.

Lighthill never received a Ph.D. degree from any university. However, he was awarded honorary doctorate degrees from at least 24 universities including Liverpool (1961), Leicester (1965), Strathclyde (1966), Princeton (1966), Essex (1967), East Anglia (1968), Manchester (1968), Bath (1969), St. Andrews (1969), Surrey (1969), Cranfield (1974), Paris (1975), Aachen (1975), Rensselaer (1980), Leeds (1983), Brown (1984), Southern California (1984), Lisbon (1986), Rehovot (1987), London (1993), Compiègne (1994), Kiev (1994), St. Petersburg (1996) and Florida State (1996).

Many students of the University of Manchester, Imperial College and the University of Cambridge including M. R. Abbott, J. R. Blake, R. W. Blake, H. Cameron, G. D. Crapper, E. Cumberbatch, S. N. Curle, J. M. Fitz-Gerald, N. C. Freeman, J. H. Gerrard, F. A. Goldsworthy, I. M. Hall, G. J. Hancock, T. K. Herring, M. S. Howe, A. I. Mees, J.J.L. Higdon, P. E. Rapp, J.M.V. Rayner, N. Riley, M. J. Simon, F. B. Smith, M. A. Swinbanks, E. J. Varley, and G. B. Whitham have completed their research or Ph.D. thesis under Lighthill's supervision. Subsequently, almost all of them did much more original work in applied mathematics, fluid dynamics and biofluiddynamics and became famous for their outstanding contributions.

After his formal retirement from the University College London in 1989, he was appointed Emeritus Scientist by the College. Sir Eric Ash (1989) describes Sir James Lighthill's contributions to the University College London as Provost:

"James Lighthill was indeed a brilliant scientist; but he was also a polymath, with knowledge, insight and enthusiasm for the arts and humanities. He would invariably take the chair at inaugural lectures and, in thanking the speaker, provide an erudite coda — for any discipline — be it Egyptology, literature (illuminated by his ability to read in most modern European languages), medicine, or our own field of engineering. He was able to inspire his colleagues over the whole range of academic disciplines. Without the slightest doubt, during his watch, Lighthill succeeded in raising academic standards and in enhancing the international recognition accorded to University College."

Even after his retirement, Lighthill continued his research, publications, presentations, and professional service to national and international scientific community. He accepted the Royal Society Lectureship to deliver the Humphry Davy Lecture in 1991 and the Inaugural Perkins Memorial Lecture in 1995. In October of the same year, Sir James and Lady Nancy visited the University of Central Florida (UCF) for a week to deliver three lectures on fluid mechanics as part of the Distinguished Lecture Series in Mathematics at UCF. In 1993, he received the ICASE/LARC Theodorsen Lectureship Award.

In November 1996, Sir James and Lady Nancy came to Florida State University at Tallahassee to attend the International Symposium on Theoretical and Computational Fluid Mechanics which was organized in honor of Sir James Lighthill to celebrate his monumental contributions to fluid mechanics, applied mathematics, and the scientific community of the world. In 1997, the *Collected Papers of Sir James Lighthill* was published by the Oxford University Press. As editor of Sir James' collected papers M. Hussaini states the following in his General Introduction: "For more than half a century, his contributions spanned the fields of aeronautics, astrophysics, atmospheric and oceanographic sciences, and biofluid dynamics. But Lighthill is distinguished not just by the diversity and excellence of his technical contributions; his unique vision and perspicacity are evident in the groundbreaking, original work in every field he touched. In certain areas of aerodynamics and biofluid dynamics — such as wave propagation, aeroacoustics, and animal motion — his first articles were seminal and remain virtually the last word on the subject."

After the symposium at Tallahassee, Sir James and his wife returned to England. Sir James remained at the University College London until his death on July 17, 1998 after completing a nine-hour swim around the Channel Island of Sark against high tides and strong winds. The 1998

Copley Medal of the Royal Society (the highest award of the Royal Society) has been awarded posthumously to Sir James Lighthill.

Sir James Lighthill was undoubtedly one of the most brilliant and influential fluid dynamists of the twentieth century. He revolutionized applied mathematics with his remarkable contributions to modern fluid dynamics. There is no doubt at all about Lighthill's profound and everlasting impact on mathematical sciences and the scientific community of the world. His lifelong concern for quality mathematics instruction at national and international levels, and for inevitable loss by natural hazards reveals the unique character of this man. He will be remembered forever not only for his great scientific achievements, but also his unique contribution to the welfare of the human race. In many ways, Sir James was the epitome of the applied mathematical community.