

Contents

| | |
|---|-----|
| <i>Preface</i> | vii |
| 1. Conjugacy Classes, Characters, and Clifford Theory | 1 |
| 1.1 Class Functions and Characters | 1 |
| 1.2 Induced and Tensor-induced Modules | 3 |
| 1.3 Schur's Lemma | 4 |
| 1.4 Brauer's Permutation Lemma | 6 |
| 1.5 Algebraic Conjugacy | 7 |
| 1.6 Coprime Actions | 9 |
| 1.7 Invariant and Good Conjugacy Classes | 10 |
| 1.8 Nonstable Clifford Theory | 12 |
| 1.9 Stable Clifford Theory | 13 |
| 1.10 Good Conjugacy Classes and Extendible Characters | 18 |
| 2. Blocks of Characters and Brauer's $k(B)$ Problem | 19 |
| 2.1 Modular Decomposition and Brauer Characters | 19 |
| 2.2 Cartan Invariants and Blocks | 21 |
| 2.3 Defect and Defect Groups | 23 |
| 2.4 The Brauer–Feit Theorem | 25 |
| 2.5 Higher Decomposition Numbers, Subsections | 26 |
| 2.6 Blocks of p -Solvable Groups | 28 |
| 2.7 Coprime $\mathbb{F}_p X$ -Modules | 31 |
| 3. The $k(GV)$ Problem | 32 |
| 3.1 Preliminaries | 32 |
| 3.2 Transitive Linear Groups | 34 |
| 3.3 Subsections and Point Stabilizers | 36 |
| 3.4 Abelian Point Stabilizers | 41 |

| | |
|--|-----|
| 4. Symplectic and Orthogonal Modules | 45 |
| 4.1 Self-dual Modules | 45 |
| 4.2 Extraspecial Groups | 47 |
| 4.3 Holomorphs | 49 |
| 4.4 Good Conjugacy Classes Once Again | 54 |
| 4.5 Some Weil Characters | 56 |
| 4.6 Symplectic and Orthogonal Modules | 60 |
| 5. Real Vectors | 63 |
| 5.1 Regular, Abelian and Real Vectors | 63 |
| 5.2 The Robinson–Thompson Theorem | 66 |
| 5.3 Search for Real Vectors | 68 |
| 5.4 Clifford Reduction | 71 |
| 5.5 Reduced Pairs | 74 |
| 5.6 Counting Methods | 74 |
| 5.7 Two Examples | 77 |
| 6. Reduced Pairs of Extraspecial Type | 82 |
| 6.1 Nonreal Reduced Pairs | 82 |
| 6.2 Fixed Point Ratios | 84 |
| 6.3 Point Stabilizers of Exponent 2 | 86 |
| 6.4 Characteristic 2 | 90 |
| 6.5 Extraspecial 3-Groups | 92 |
| 6.6 Extraspecial 2-Groups of Small Order | 96 |
| 6.7 The Remaining Cases | 103 |
| 7. Reduced Pairs of Quasisimple Type | 110 |
| 7.1 Nonreal Reduced Pairs | 110 |
| 7.2 Regular Orbits | 112 |
| 7.3 Covering Numbers, Projective Marks | 115 |
| 7.4 Sporadic Groups | 119 |
| 7.5 Alternating Groups | 121 |
| 7.6 Linear Groups | 125 |

| | | |
|------|---|-----|
| 7.7 | Symplectic Groups | 129 |
| 7.8 | Unitary Groups | 136 |
| 7.9 | Orthogonal Groups | 145 |
| 7.10 | Exceptional Groups | 147 |
| 8. | Modules without Real Vectors | 148 |
| 8.1 | Some Fixed Point Ratios | 148 |
| 8.2 | Tensor Induction of Reduced Pairs | 149 |
| 8.3 | Tensor Products of Reduced Pairs | 155 |
| 8.4 | The Riese–Schmid Theorem | 156 |
| 8.5 | Nonreal Induced Pairs, Wreath Products | 160 |
| 9. | Class Numbers of Permutation Groups | 170 |
| 9.1 | The Partition Function | 170 |
| 9.2 | Preparatory Results | 171 |
| 9.3 | The Liebeck–Pyber Theorem | 172 |
| 9.4 | Improvements | 174 |
| 10. | The Final Stages of the Proof | 180 |
| 10.1 | Class Numbers for Nonreal Reduced Pairs | 180 |
| 10.2 | Counting Invariant Conjugacy Classes | 182 |
| 10.3 | Nonreal Induced Pairs | 185 |
| 10.4 | Characteristic 5 | 186 |
| 10.5 | Summary | 194 |
| 11. | Possibilities for $k(GV) = V $ | 195 |
| 11.1 | Preliminaries | 195 |
| 11.2 | Some Congruences | 197 |
| 11.3 | Reduced Pairs | 199 |
| 12. | Some Consequences for Block Theory | 202 |
| 12.1 | Brauer Correspondence | 202 |
| 12.2 | Clifford Theory of Blocks | 203 |
| 12.3 | Blocks with Normal Defect Groups | 207 |

| | |
|---|-----|
| 13. The Non-Coprime Situation | 209 |
| Appendix A: Cohomology of Finite Groups | 213 |
| Appendix B: Some Parabolic Subgroups | 217 |
| Appendix C: Weil Characters | 221 |
| Bibliography | 225 |
| List of Symbols | 230 |
| Index | 231 |