

PREFACE TO THE FIRST EDITION

The term *chaos* is often used to describe the phenomena in which the system's trajectories are sensitive to the slightest changes in initial conditions. In reality, the properties of such motion resemble those of random motion. If one restricts oneself to Hamiltonian systems with area-preserving dynamics, the above definition of chaos would appear perplexing. Can a trajectory be chaotic at times and "regular" for the rest of the time? How does a regular trajectory transform itself into a chaotic one? And what type of randomness characterises chaotic dynamics? Fortunately, clear definitions exist for regular and chaotic motions. Conditionally periodic motion is an example of regular dynamics. The examples of "ideal" chaotic motion refer to the motion on the negative curvature surface and the so-called Anosov systems. However, real physical systems or their simplified models are very different from the ideal models on chaos. A good example is that of a pendulum disturbed by a periodic (non-random) force.

Our understanding of chaos is fraught with the following difficulties. The motion known as chaotic occupies a certain area (called stochastic sea) in the phase space. In ideal chaos, the stochastic sea is occupied in a uniform manner. This is, however, not the case in real systems or models. The phase space contains many "islands" which a chaotic trajectory cannot penetrate. Initially, it had appeared that the effects of islands could be easily accounted for by simply changing the phase volume of the stochastic sea. We now believe that this is not true and the important properties of chaotic dynamics are in fact determined by the properties of the motion near the boundary of islands.

The difficulties in understanding Hamiltonian chaos can also be described in an informal way. While regular and chaotic motions possess some degree of uniformity (monotonicity), which is used in their definitions, real chaotic motion boasts intermediate properties (between regular and chaotic motions) that have not been accurately defined and formulated. Therefore, we can neither use the KAM theory (as the conditions of non-degeneracy are violated in most cases) nor the Sinai's method of Markov partitions or related techniques (because the correlations do not decay exponentially and the Markov property is violated in some areas) and the estimates of Arnold diffusion (since a much faster diffusion takes place).

This book considers many of the difficulties described above in analysing Hamiltonian chaos in real systems. The reader is treated to the unconventional application of the fractal dimension to space-time objects, different versions of the renormalisation group method, fractional kinetics, and Poincaré recurrences theory as well as the more traditional applications of the Poincaré and separatrix maps.

This book is useful to the reader who has an undergraduate degree in physics. It does not include any methods that are beyond the standard mathematical physics techniques except for some fractional calculus which is provided in a special appendix. It is useful to physicists, engineers and those who are interested in the current problems in chaos theory and its applications. While mathematicians will not be able to find any rigorously proven results here, they will learn about the challenges of "real physics". Towards that end, we have included many examples of numerical simulations. Some of the examples were extracted (and updated) from the author's previous works. The material covered is based partly on the course offered by the author at the Courant Institute of Mathematical Sciences and the Department of Physics in New York University. Much of it, however, is new and can be used as a source of information on the new and emerging directions in modern chaos theory applied to physical problems.

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PREFACE TO THE SECOND EDITION

After the first publication of the book there were different developments in the theory and experiments related to a highly complex intermittent character of chaotic dynamics and to the fractional structure of dynamics and kinetics. To reflect these changes, the second edition of the book includes different new sections and minor additions in corresponding places. There are extensions of the sections related to Maxwell's Demon and billiards. New sections on Ballistic Mode Islands, Rhombic Billiards, Persistent Fluctuations, and Log-Periodicity can be found. A new Chapter 12 on weak chaos and pseudochaos is also added. Pseudochaos is introduced as random dynamics with zero Lyapunov exponent. As an application of the pseudochaos is the description of fractional kinetics along the Filamented Surfaces that has numerous applications in magneto- and hydrodynamics. Appendices have been considerably modified to extend a reference material related to basic formulas of fractional calculus.

Different misprints and typos of the first edition are eliminated in the second edition. New sections and references are marked by (*). It is my great pleasure to thank Mark Edelman for his help in preparing the material and figures for the second edition.