Preface

It was in the early 1990s that I first did some work in the area of planar Stokes flows. At that time I had no idea that I would be working in the area for the following dozen years, or that I would contemplate writing a book dealing with slow viscous flows. Soon thereafter, in correspondence with Professor A. Acrivos I was convinced by him of the importance of and the need for fully three-dimensional work, which I then took up. I soon found out that there were no fully three-dimensional internal Stokes flow solutions available in the literature at that time; when I started work on these I found out how hard they were to obtain. Joint work with Dr. R. Kidambi on satellite fuel tank problems led to our examining unsteady viscous flows and the problems of laminar mixing by chaotic advection. And at a conference at the University of Hyderabad, a talk by Professor E. Chadwick led me to the realization that external flows could be handled in the same uniform way that we had for internal flows. It was now clear that there was a sizable and coherent body of work that was of interest and that could well be put together as a book.

In the meanwhile, there was a growing awareness that although the method of eigenfunction expansions was classical and well known, its application to non-self-adjoint problems was in fact very limited. And almost invariably the use was of expansions in real, orthogonal eigenfunctions. The only place that I am aware of where an example of complex eigenfunctions is given is in Bernard Friedman's excellent book (Friedman 1956); and even here the operator is self-adjoint but the boundary conditions are non-canonical. Hundreds of boundary value problems are solved in Morse & Feshbach (1953), Titchmarsh (1958), Carslaw & Jaeger (1959) and Finlayson (1972), and more recently in Hassani (1999) and

Gustafson & Wilcox (1999). But it appears that not a single complex valued expansion is considered, nor one where the coefficients are determined by something other than an orthogonality or biorthogonality property of the eigenfunctions. My own experience with Stokes flows is that more often than not, the natural eigenfunctions are complex and there is usually no convenient inner product available. We have, nevertheless, been able to solve many difficult problems. So it seemed worthwhile to document and show how non-classical, complex eigenfunction expansions can be used to efficiently solve problems in fields in which they naturally arise.

There is very little overlap between this book and the earlier treatises of Happel & Brenner (1965) and Langlois (1964), or even the more recent texts of Kim & Karilla (1991) and Pozrikidis (1992). All of these deal almost exclusively with external flows with the main applications being to problems connected with suspensions, sedimentation and the motion of liquid drops. The main emphasis in this book is on internal flows. External flows are also dealt with but in the framework of the Oseen equations, whereas the cited references deal mostly in the framework of the Stokes equations. Thus there is hardly any common material. However, I recall reading in one of Graham Greene's essays: "Kingslake once referred to 'that nearly immutable law which compels a man with a pen in his hand to be uttering every now and then some sentiment which is not his own', and compared an author with a French peasant under the old regime, bound to perform a certain amount of work upon the public highways." I have tried to keep my workload on the public highways to as little as possible.

A potential reader would want to know what the special features of this book are. Much of the material in this book is new, having been published only very recently in the journal literature. And some of the material has not so far been published anywhere. There are discussions here of the resolution of the corner singularity problem with discontinuous data (Bloor & Wilson 2006), of three-dimensional Moffatt eddies in the cone (Malyuga 2005, Shankar 2005c), of three-dimensional wedge flows (Sano & Hasimoto 1980, Moffatt & Mak 1999, Shankar 2000), of oscillatory internal flows (Shankar, Kidambi & Hariharan 2003, Branicki & Moffatt 2006), of three-dimensional natural convection (Shankar 2005d), and much more. Of the material that has so far not been published before, the most important are (i) the analytical solution of the three-dimensional flow in a rectangular container including the extension to three dimensions of the Papkovich–Fadle eigenfunctions, (ii) the analysis of mixing in a rectangular container including comparisons with experimental data, and (iii) the analysis of external

Oseen flows past arbitrary bodies using eigenfunction expansions and least squares. There is also the advantage that all these topics are treated by basically the same techniques and with all the background material given between the same two covers.

A decision had to be made when writing the book as to what level it should be aimed at. From the response to our Annual Review article, I gathered that there was considerable interest in slow flows over a wide spectrum of engineers and scientists, many of whom may not have had a strong background in fluid mechanics; and who may not have been interested in details but just wanted to know about the qualitative features of these flows. It appeared that there was a case for books at two levels at least. But in the present economic and publishing environment this would be impossible. I therefore made the conscious decision to write a book that would try to cater to all these needs: for an elementary text, for a handbook of slow flows and for a research monograph. As a consequence the level of the book varies from elementary to advanced, with chapters of widely different lengths and with a large number of figures. My aim is that a researcher, say a chemist or biologist, who does not normally deal in fluid mechanics should be able to at least get a qualitative feel from just studying the figures and captions. A graduate student should be able to learn about the complex eigenfunction method in all its details from just reading a few chapters; enough to be able to solve his or her own problem in another field. Such a reader would also find the codes on the accompanying CD of use. The codes should also be useful to students who can use them to visualize complex flow fields. And I should be gratified if a fluid dynamicist found the discussion of three-dimensional corner eddies and of mixing interesting and enjoyable. These are my hopes and aspirations; but it is possible that in aiming for all these I satisfy none of the groups I am aiming at. But this would have followed from a conscious decision and not by accident.

It was while working on this book that I discovered the 'embedding method' for solving boundary value problems in non-simple, non-canonical domains. I have since found out that the phrase 'embedding method' is used in other contexts, especially in finite-element numerical solutions for complex geometries, where it is also known as the 'fictitious domain' method. However, apart from the common use of embedding domains, the methods have little in common: here, complete sets of global complex eigenfunctions are used to determine the solution, with the solution contained in a comparatively small number of complex scalars. This is a very powerful method for solving problems in practical geometries that can even be applied to

certain nonlinear problems, provided the field equations are linear. This is the first book that explains this method in some detail and provides all the necessary background. This material should be useful to practitioners in other fields of science and engineering.

There are many people who have helped me during the preparation of this book but there are two who need special mention right here in the preface. Through his insightful and clearly written papers, his exciting lectures, and his critical but helpful comments as a referee, Professor Keith Moffatt, F.R.S., has been a source of inspiration. Without his active support this book would not have come into being. For all these and for kindly agreeing to write a foreword to this book, I am most grateful to him. Thank you, Keith. It was my accidental good fortune that Dr. R. Kidambi walked into my office at NAL in 1999 looking for a job. Since then we have had an excellent time learning a lot together in a great hurry. As important have been the good laughs that we have had, as have our long discussions on the Eightfold Path and Vipassana. Ranga also most kindly agreed to write two important chapters of this book, Chaps. 16 and 17, based on his own work. Ranga, thank you.

Finally, I am greatly indebted, as always, to my wife, Priti, and children, Mridula, Nachiket and Niveditha for their love and support. Mridula, especially has asked me not to write any more books. To the extent that this book has any merit, it is dedicated to them with love.

P. N. Shankar
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