Preface

Stochastic modelling has come to play an important role in many branches of science and industry. An area of particular interest has been the automatic control of stochastic systems, with consequent emphasis being placed on the analysis of stability in stochastic models.

The hybrid systems driven by continuous-time Markov chains have been used to model many practical systems where they may experience abrupt changes in their structure and parameters caused by phenomena such as component failures or repairs, changing subsystem interconnections, and abrupt environmental disturbances. The hybrid systems combine a part of the state that takes values continuously and another part of the state that takes discrete values. In 1971, [Kazangey and Sworder (1971)] presented a jump system, where a macroeconomic model of the national economy was used to study the effect of federal housing removal policies on the stabilization of the housing sector. The term describing the influence of interest rates was modelled by a finite-state Markov chain to provide a quantitative measure of the effect of interest rate uncertainty on optimal policy. [Athans (1987)] suggested that the hybrid systems would become a basic framework in posing and solving control-related issues in Battle Management Command, Control and Communications (BM/C^3) systems. The hybrid systems were also considered for the modelling of electric power systems by [Willsky and Rogers (1979)] as well as for the control of a solar thermal central receiver by [Sworder and Robinson (1973)]. In his book, [Mariton (1990)] explained that the hybrid systems had been emerging as a convenient mathematical framework for the formulation of various design problems in different fields such as target tracking (evasive target tracking problem), fault tolerant control and manufacturing processes.

One of the important classes of the hybrid systems is the linear jump systems

$$\dot{x}(t) = A(r(t))x(t)$$

which have been investigated for more than 30 years. The generalisation of the linear jump systems is the stochastic differential equations (SDEs) with Markovian switching

$$dx(t) = f(x(t), r(t))dt + g(x(t), r(t))dB(t),$$

which have recently received a great deal of attention. Here the state vector has two components $x(t)$ and $r(t)$: the first one is in general referred to as the state while the second one is regarded as the mode. In its operation, the system will switch from one mode to another in a random way, and the switching between the modes is governed by a Markov chain. The study of hybrid systems has included the optimal regulator, controllability, observability, stability and stabilization *etc.* For more information on the hybrid systems the reader is referred to [Basak *et al.* (1996); Bouks (1993); Costa and Boukas (1998); Dragan and Morozan (2002; Feng *et al.* (1992); Ji and Chizeck (1990); Lewin (1986); Mao *et al.* (2000); Morozan (1998); Pan and Bar-Shalom (1996); Souza and Fragoso (1993); Shaikhet (1996); Skorohod (1989); Sworder and Robinson (1973); Wonham (1970)]. In particular, the authors have made significant contributions to this area (see e.g. [Mao (1999a); Mao (1999b); Mao (2000b); Mao (2002b); Mao *et al.* (2000)] and [Yuan and Mao (2003a)]–[Yuan *et al.* (2003)]).

Although there is a book [Mariton (1990)] on the linear jump systems, there is so far no book on SDEs with Markovian switching and the papers on them were published in different journals in a form which is not convenient for the reader to understand the theory systematically. This book was therefore written. Some important features of this text are as follows:

- The text will be the first systematic presentation of the theory of SDEs with Markovian switching. It will present the basic principles of SDEs with Markovian switching at an introductory level but will emphasise the current research trends in the field of SDEs with Markovian switching at an advanced level.
- The text will cover various types of equations with Markovian switching from stochastic differential equations through interval systems to stochastic functional differential equations, all with Markovian switching.

- This text will discuss a number of approximation schemes including the Euler–Marayama and Carathedory under both global and local Lipschitz condition. Especially the numerical methods for SDEs under local Lipschitz condition are currently a very hot topic.
- This text will demonstrate the manifestations of the general Lyapunov method by showing how this effective technique can be adopted to study entirely different qualitative and quantitative properties of stochastic systems, e.g. asymptotic bounds, exponential stability and invariant measures.
- This text will emphasise the analysis of stability which is vital in the automatic control of stochastic systems.
- This text will be mainly based on the authors' recent research papers, for example, [Mao (1999a); Mao (1999b); Mao (2000b); Mao (2002b); Mao *et al.* (2000)] and [Yuan and Mao (2003a)]–[Yuan *et al.* (2003)]. It will hence discuss many hot topics including various applications to finance, population dynamics and control.

In other words, the text will take all the features of Itô equations, Markovian switching, interval systems as well as time-lag into account. The theory developed will be applicable in different and complicated situations in many branches of science and industry. In particular, we will discuss a number of important applications including population dynamics, financial modelling, stochastic stabilization and stochastic neural networks. All of them are currently hot topics in research.

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