Chapter 1

Batch Melting

1.1. Overview of Melting Models

Partial melting of Earth and planetary materials is a fundamental process that contributes to the differentiation and evolution of the Earth. Modeling of partial melting using trace element concentrations is often required to understand the melt generation and segregation process and to interpret the chemical composition of primary melts. The behaviors of trace elements during partial melting present a good problem for the application of mathematics.

There are three general models based on the extent of chemical equilibrium between the solid and melt: batch, fractional (Schilling and Winchester, 1967; Gast, 1968; Shaw, 1970) and dynamic melting (Langmuir et al., 1977; McKenzie, 1985; Zou, 1998; Zou, 2000; Zou and Reid, 2001). Another model is called continuous melting (Williams and Gill, 1989; Albarede, 1995) or critical melting (Maaløe, 1982; Sobolev and Shimizu, 1992). Although continuous melting is commonly distinguished from dynamic melting in that in the former an excess melt is removed from a static column whereas in the latter the entire melting region migrates and new fertile material is added to the column, the real difference between them is only the aggregation time required to produce the magmas. The difference in aggregation time certainly affects the activity of a short-lived radioactive nuclide in magmas, however, it will not affect the concentration of stable trace elements (Williams and Gill, 1989). Therefore, although continuous melting and dynamic melting appear different conceptually, for the purpose of mathematical treatment of stable trace element fractionation, they are mathematically identical.

Among the three general models, the batch melting model assumes that melt remains in equilibrium with the solid throughout the melting event whereas the fractional melting model assumes that (1) the melt is removed from the initial source as it is formed, (2) only the last drop of melt is in equilibrium with the residue, and (3) there is no residual melt. Dynamic melting involves the retention of a critical fraction of melt in the mantle residue. During dynamic melting, when the melt mass fraction in the residue is less than the critical value for melt separation (or the critical mass porosity of the residue, Φ), there is no melt extraction (as in batch melting); when the melt fraction in the residue is greater than Φ, any infinitesimal excess melt will be extracted from the matrix.

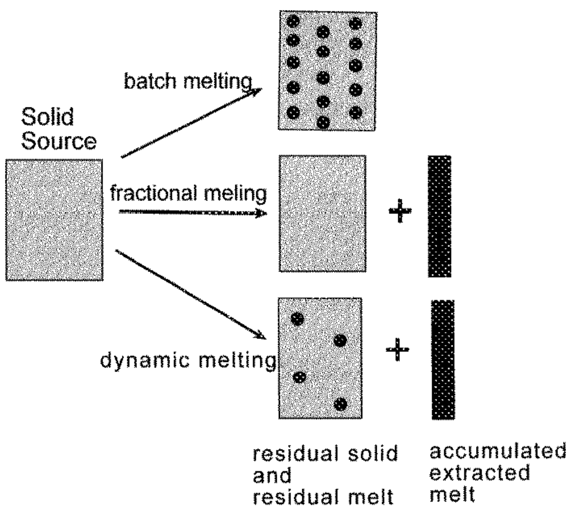

Fig. 1.1. A sample diagram showing batch, fractional and dynamic melting models. Filled circles represent melts in equilibrium with the residual solid. There is no residual melt for the fractional melting model.

The difference between the three basic models can be illustrated in the mass porosity of the melting residue (Ψ) vs. the total partial melting degree (F) diagram (Fig. 1.1). For batch melting, the mass porosity is equal to the partial melting degree until extraction of melt begins, that is, $\Psi = F$ before melt extraction begins and $\Psi = 0$ after melt extraction takes place; for (perfect) fractional melting, $\Psi = 0$ during the whole melting process; for dynamic melting, $\Psi = F$ when $F < \Phi$; $\Psi = \Phi$ when $F > \Phi$.

It is noted that the relationship between X and F is also important to distinguish the dynamic melting model from both the batch melting and the fractional melting models (Fig. 3).

For batch melting,

$$X = 0,$$

before melt extraction begins, and

$$X = F,$$

after melt extraction takes place;

for fractional melting,

$$X = F;$$

and for dynamic melting,

$$X = 0, \text{ when } F < \Phi,$$

and,

$$X = \frac{F - \Phi}{1 - \Phi}, \tag{1.1}$$

when $F > \Phi$ (Zou, 1998).

The slope in the X vs. F diagram for dynamic melting is $1/(1-\Phi)$ and is greater than 1 because $0 < \Phi < 1$.

In this chapter, we focus on the batch melting model where melt maintains chemical equilibrium with solid, and stays with solid until the final extraction.

Define F as the degree of partial melting, that is, the mass ratio of the melt over the initial mass before melting. The balance of the total mass and the mass of an element can be presented in Table 1.1.

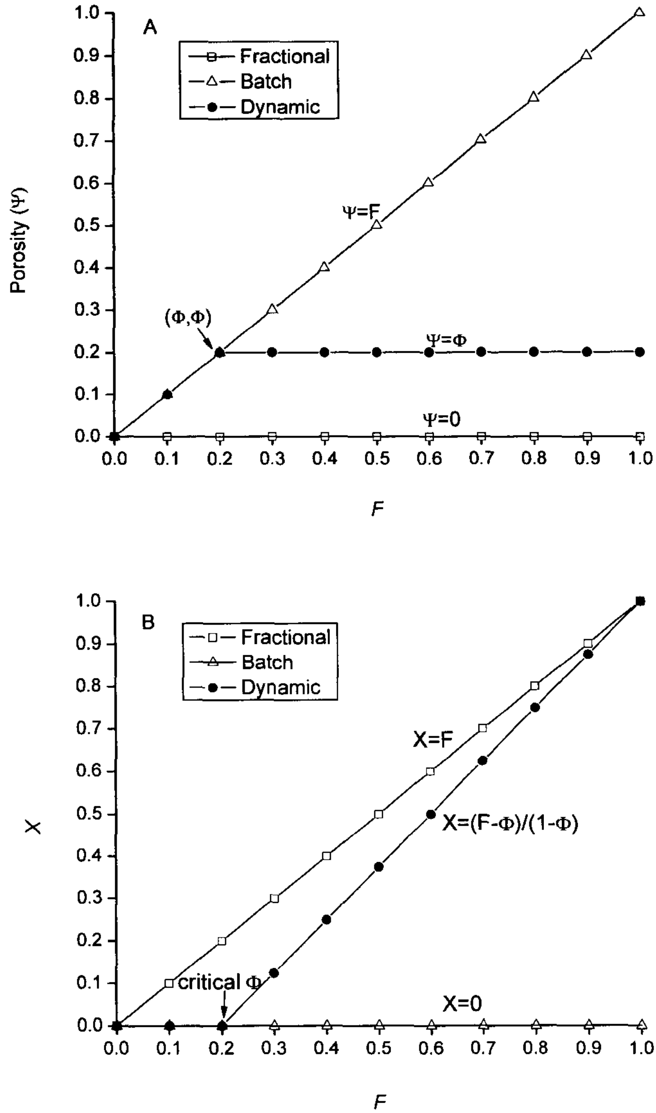

Fig. 1.2. (a) Mass porosity of melting residue (Ψ) vs. the total partial melting degree (F) diagram for batch, fractional, and dynamic melting models. (b) The fraction of extracted melt relative to the initial solid before melting starts (X) vs. the total partial melting degree (F) diagram for the three general models. Critical porosity (Φ) is set at 0.2 for illustration purpose.

Table 1.1. Mass balance during batch melting. The two-way arrows indicate interaction and equilibrium between solid and melt.

	Source	Solid		Equilibrium Melt
Conservation of total mass	M_0	$M_0(1-F)$	$\leftrightarrow$	M_0F
Conservation of the mass of an element	C_0M_0	$C_sM_0(1-F)$	$\leftrightarrow$	C_LM_0F

Mass balance requirement gives

$$C_LFM_0 + C_s(1-F)M_0 = C_0M_0 . \quad (1.2)$$

The concentration of the melt (C_L) in equilibrium with the a multiple-phase solid is related to the concentration in the solid (C_s) by

$$C_s = DC_L , \quad (1.3)$$

where D is the bulk partition coefficient

$$D = \sum x^i K^i , \quad (1.4)$$

where x^i is the mineral proportion in the residue and K^i is the mineral/melt partition coefficient.

Combination of Eq. (1.2) with (1.3) yields

$$C_L = \frac{C_0}{F + D(1-F)} . \quad (1.5)$$

This is the fundamental equation for batch melting. More complex batch melting models deal with the changes of the bulk partition coefficient D. Although this formula may also be expressed as

$$C_L = \frac{C_0}{D + F(1-D)} . \quad (1.6)$$

Equation (1.5) is convenient if we need to consider the variation of D.

Example. For a three mineral phase with 50% olivine (ol), 30% clinopyroxene (cpx) and 20% orthopyroxene (opx), if the mineral/melt partition coefficients are $K^{ol} = 0.01$, $K^{cpx} = 0.2$ and $K^{opx} = 0.08$, the bulk partition coefficients between melt and the multi-phase solid is

$$D = \sum x^i K^i = x^{ol} K^{ol} + x^{cpx} K^{cpx} + x^{opx} K^{opx}$$
$$= 0.5 \times 0.01 + 0.4 \times 0.2 + 0.1 \times 0.08 = 0.093.$$

1.2. Modal Batch Melting

During modal melting, the melting proportion is the same as the mineral source proportion and thus x^i is constant throughout the melting process $x^i = x_0^i$. When K^i is also constant ($K^i = K_0^i$), then the bulk partition coefficient D remains constant, Eq. (1.5) becomes

$$C_L = \frac{C_0}{F + D_0(1-F)} = \frac{C_0}{D_0 + F(1-D_0)}, \quad (1.7)$$

where

$$D_0 = \sum x_0^i K_0^i. \quad (1.8)$$

Example. Calculate C_L/C_0 for three elements with bulk partition coefficients of 0.02, 0.10 and 1.5, respectively. From Eq. (1.7) we can make Fig. 1.3 by plotting C_L/C_0 vs. F for the three D values.

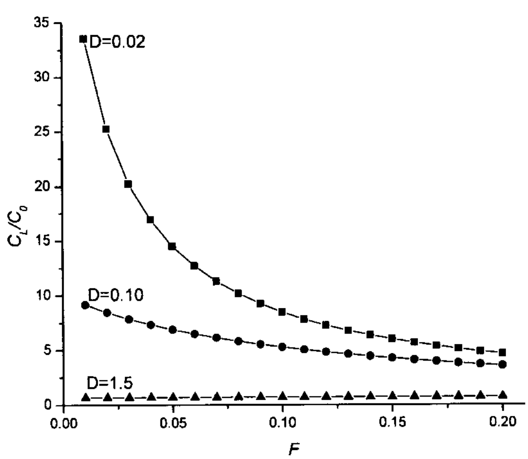

Fig. 1.3. Variation of Source-normalized concentrations with the degree of artial melting for trace elements with different bulk partition coefficients.

1.3. Nonmodal (Eutectic) Batch Melting

Modal melting normally does not happen. Due to preferential melting of some minerals in the source rock, x^i must change. For example, a mantle rock may have 10% cpx, but the cpx may contribute 25% of the melt as a result of preferential melting of cpx, resulting the decrease of x^{cpx} as a function of F during melting.

The mass conservation for the phase i gives

$$M_0(1-F)x^i + M_0Fp^i = M_0x_0^i, \tag{1.9}$$

where p^i is the fractional contribution of phase i to the melt, x_0^i is the initial fraction of phase i in the source, $M_0(1-F)$ is the mass of the residual solid, and M_0F is the mass of the melt. Equation (1.9) may be re-expressed as

$$x^i = \frac{x_0^i - Fp^i}{1-F}. \tag{1.10}$$

Note that for modal melting, melting proportion is the same as the initial mineral proportion in the source ($p^i = x_0^i$). In this case, Eq. (1.10) reduces to

$$x^i = x_0^i. \tag{1.11}$$

1.3.1. Eutectic batch melting with constant K^i

When K^i is a constant, the bulk partition coefficient becomes

$$D = \Sigma x^i(F)K^i = \Sigma\frac{x_0^i - Fp^i}{1-F}K_0^i = \frac{\sum x_0^i K_0^i - F\sum p^i K_0^i}{1-F}. \tag{1.12}$$

Defining

$$P = \sum p^i K_0^i, \tag{1.13}$$

we have

$$D = \frac{D_0 - PF}{1-F}. \tag{1.14}$$

Differentiation of D with respect to F gives

$$dD/dF = (D_0 - P_0)/(1-F)^2 . \tag{1.15}$$

If $D_0 < P_0$, then $dD/dF < 0$ and D decreases as melting proceeds; and if $D_0 > P_0$, then $dD/dF > 0$ and D increases during melting (Fig. 1b). In fact, the relationship $D_0 < P_0$ implies the preferential melting of the minerals with high mineral/melt distribution coefficients and thus the solid residue is left with higher proportions of minerals of low distribution coefficients; the opposite is true for the case where $D_0 > P_0$.
Substitution of (1.14) into (1.5) results in

$$C_L = \frac{C_0}{D_0 + F(1-P)} . \tag{1.16}$$

Equation (1.16) is the famous non-modal batch melting equation from Shaw (1970).

1.3.2. Eutectic batch melting with linear change of K^i

We then expand $K^i(F)$ in a Taylor series and only retain terms up to the first order

$$K^i(F) = K_0^i + a^i F . \tag{1.17}$$

The above simple assumption of linear variation of $K^i(F)$ as a function of F is at least appropriate for evaluating the major effects of decreasing or increasing distribution coefficients (Greenland, 1970; Hertogen and R., 1976). More complicated forms of the distribution coefficients as a function of F can also be assumed; however, they involve more parameters and do not necessarily better describe the variations in distribution coefficients.
Substitution of (1.17) and (1.10) into (1.4) gives

$$D = \Sigma x^i(F) K^i(F) = \Sigma \frac{x_0^i - Fp^i}{1-F}\left(K_0^i + a^i F\right), \tag{1.18}$$

or

$$D=\frac{1}{1-F}\left[-\left(\Sigma a^i p^i\right)F^2+\left(-P_0+\Sigma a^i x_0^i\right)F+D_0\right]. \quad (1.19)$$

Substitution of (1.19) into (1.5) gives

$$\begin{aligned} C_L &= \frac{C_0}{F+D(1-F)} \\ &= \frac{C_0}{D_0+\left(1-P_0+\sum a^i x_0^i\right)F-\left(\sum a^i p^i\right)F^2}. \end{aligned} \quad (1.20)$$

1.3.3. Eutectic batch melting with linear change of K^i, linear change of p^i

If p^i is also a function of F, then Eq. (1.9) for mass conservation of phase i should be replaced by

$$x^i\left(1-F\right)=x_0^i-\int_0^F p^i dF, \quad (1.21)$$

We may assume linear variations in p^i during nonmodal melting

$$p^i = p_0^i + b^i F, \quad (1.22)$$

where $b^i=\left(p^i(F_m)-p_0^i\right)/F_m$, F_m is the maximum degree of melting when one of the phases melt completely, and $p^i(F_m)$ is the net fractional contribution to the melt when the degree of partial melting is F_m. By substituting Eq. (1.22) into Eq. (1.21), we have

$$x^i = \frac{x_0^i - Fp_0^i - 0.5b^i F^2}{1-F}. \quad (1.23)$$

Combination of Eq. (1.23), (1.17) and (1.4) leads to

$$D=\sum\frac{x_0^i - p_0^i F - 0.5b^i F^2}{1-F}\left(K_0^i + a^i F\right), \quad (1.24)$$

or

$$D=\frac{1}{1-F}\left(A_0F^3+A_1F^2+A_2F+D_0\right), \quad (1.25)$$

where

$$A_0 = -\left(\sum 0.5a^i b^i\right), \tag{1.26}$$

$$A_1 = -\sum a^i p_0^i - \sum 0.5b^i K_0^i, \tag{1.27}$$

$$A_2 = \sum a^i x_0^i - \sum p_0^i K_0^i = \sum a^i x_0^i - P_0, \tag{1.28}$$

$$D_0 = \sum K_0^i x_0^i. \tag{1.29}$$

Substitution of (1.25) into (1.5) yields

$$C_L = \frac{C_0}{D_0 + (1 + A_2)F + A_1 F^2 + A_0 F^3}. \tag{1.30}$$

1.4. Incongruent Batch Melting

Eutectic melting generates melt only, however, melting reactions in the mantle and crust often produce not only melt but also minerals. During incongruent melting, x^i changes all the time. But it varies in a different manner as compared with eutectic melting.

During partial melting of the crust and the mantle, some minerals melt congruently and others melt incongruently. Melting of congruent minerals only produces melt, and such a process can be expressed as

$$\theta_1 + \theta_2 + \cdots + \theta_u \rightarrow Liq_\theta, \tag{1.31}$$

where θ_1, θ_2, and θ_u are minerals that melt congruently and Liq_θ is the liquid formed by congruent melting.

By comparison, melting of incongruent minerals produces not only melt but also minerals, and such a melting reaction process can be expressed as

$$\alpha_1 + \alpha_2 + \cdots + \alpha_v \rightarrow \beta_1 + \beta_2 + \cdots + \beta_w + Liq_\alpha, \tag{1.32}$$

where α_1, α_2 and α_v represents the minerals that melt incongruently, β_1, β_2, and β_w are product minerals, and Liq_α is the liquid formed by melting reactions. As an example, quartz, biotite and sillimanite are α

minerals while is a β mineral, in the following melting reactions (in mass units) in a biotite-sillimanite-quartz gneiss

0.27 quartz + 0.43 biotite + 0.30 sillimanite = 0.72 cordierite + 0.28 melt.

As another example, at 10 kbar, melting of a spinel peridotite has the following melting reaction in mass units (Kinzler and Grove, 1992)

0.82 cpx +0.40 opx +0.08 spinel = 1.0 melt +0.30 olivine

clinopyroxene (cpx), orthopyroxene (opx) and spinel are α minerals whereas olivine is a β mineral.

In fact, a source rock often contains both congruent minerals and incongruent minerals, therefore, a general melting equation is

$$\begin{aligned}&(\theta_1+\theta_2+\cdots+\theta_u)+(\alpha_1+\alpha_2+\cdots+\alpha_v)\rightarrow\\&Liq_\theta+(\beta_1+\beta_2+\cdots+\beta_w+Liq_\alpha)\end{aligned}. \tag{1.33}$$

The minerals (β) that are produced in the melting reaction can be the congruent minerals (θ) already present in the system or minerals that are new to the system. Incongruent melting is common in the process of mantle and crust melting (Zeck, 1970; Benito-Garcia and Lopez-Ruiz, 1992; Kinzler and Grove, 1992; Kinzler, 1997; Gudfinnsson and Presnall, 1996; Walter, 1998).

Let S_θ^i and L_θ be the consumed mass of a congruent mineral and the mass of the produced melt by congruent melting, respectively. Then, according to Eq. (1.31), we have the total consumed mass of all congruent minerals as $\sum S_\theta^i = L_\theta$. Let S_α^i, S_β^i and L_α be the converted mass of an incongruent mineral, the produced mass of a mineral, and the mass of the produced melt, respectively, during incongruent melting, then, according to Eq. (1.32), we have $\sum_\alpha S_\alpha^i = \sum_\beta S_\beta^i + L_\alpha$. The mass balance for Eq. (1.33) is

$$\sum_\alpha S_\alpha^i + \sum_\theta S_\theta^i = \sum_\beta S_\beta^i + L_\alpha + L_\theta. \tag{1.34}$$

For a general melting reaction in Eq. (1.33), the degree of partial melting (F) is the mass fraction of the total melt $\left(L_\theta + L_\alpha\right)$ relative to the initial amount of the source (M_0), or,

$$F = \left(L_\theta + L_\alpha\right) / M_0 . \quad (1.35)$$

According to the definitions of S_θ^i, S_α^i, S_β^i, L_θ, and L_α in Eq. (1.33), the total mass of the converted minerals is $\left(\sum_\theta S_\theta^i + \sum_\alpha S_\alpha^i\right)$, and the total mass of the produced melt is $(L_\theta + L_\alpha)$. Therefore, the fractional contributions of phase i to the total mass of the converted minerals through congruent melting and incongruent melting are, respectively,

$$p_\theta^i = S_\theta^i \Big/ \left(\sum_\theta S_\theta^i + \sum_\alpha S_\alpha^i\right), \quad (1.36)$$

$$p_\alpha^i = S_\alpha^i \Big/ \left(\sum_\theta S_\theta^i + \sum_\alpha S_\alpha^i\right). \quad (1.37)$$

The mass fractions of incongruent minerals converted into melt or mineral i are, respectively,

$$t^l = L_\alpha \Big/ \left(\sum_\beta S_\beta^i + L_\alpha\right) = L_\alpha \Big/ \sum_\alpha S_\alpha^i , \quad (1.38)$$

$$t^i = S_\beta^i \Big/ \left(\sum_\beta S_\beta^i + L_\alpha\right) = S_\beta^i \Big/ \sum_\alpha S_\alpha^i . \quad (1.39)$$

Consequently, we have

$$\sum_\theta p_\theta^i + \sum_\alpha p_\alpha^i = 1 , \quad (1.40)$$

$$t^l + \sum_\beta t^i = 1 . \quad (1.41)$$

From (1.36), (1.37) and (1.39), we obtain the mass fraction of net converted mineral i *relative to the total converted mass* as

$$\left(S_\theta^i + S_\alpha^i - S_\beta^i\right) \Big/ \left(\sum_\theta S_\theta^i + \sum_\alpha S_\alpha^i\right) = p_\alpha^i + p_\theta^i - t^i \sum_\alpha p_\alpha^i . \tag{1.42}$$

From (1.34), (1.37), (1.39) and (1.41), we get the mass fraction of total melt *relative to the total converted mass* as

$$\begin{aligned}(L_\theta + L_\alpha) \Big/ \left(\sum_\theta S_\theta^i + \sum_\alpha S_\alpha^i\right) &= 1 - \sum_\beta S_\beta^i \Big/ \left(\sum_\theta S_\theta^i + \sum_\alpha S_\alpha^i\right) \\ &= 1 - \left(1 - t^l\right) \sum_\alpha p_\alpha^i \end{aligned} . \tag{1.43}$$

By dividing Eq. (1.42) by Eq. (1.43), we obtain the mass fraction of net converted mineral i *relative to the total produced melt*

$$q^i = \frac{S_\theta^i + S_\alpha^i - S_\beta^i}{L_\theta + L_\alpha} = \frac{p_\theta^i + p_\alpha^i - t^i \sum_\alpha p_\alpha^i}{1 - \left(1 - t^l\right) \sum_\alpha p_\alpha^i} , \tag{1.44}$$

which can be interpreted as the actual fractional contribution of mineral i to the total melt. Multiplying Eq. (1.44) by Eq. (1.35) for the definition of the degree of partial melting (F), we obtain the mass fraction of net converted mass for mineral i *relative to the initial source amount* (M_0) as

$$x_c^i = \frac{S_\theta^i + S_\alpha^i - S_\beta^i}{M_0} = F q^i = \frac{p_\theta^i + p_\alpha^i - t^i \sum_\alpha p_\alpha^i}{1 - \left(1 - t^l\right) \sum_\alpha p_\alpha^i} F . \tag{1.45}$$

Mass balance of mineral i requires

$$x_0^i = x^i (1 - F) + x_c^i . \tag{1.46}$$

Substitution of (1.45) into (1.46) yields the variation of a mineral phase proportion as

$$x^i = \frac{x_0^i - q^i F}{1 - F} , \tag{1.47}$$

where

$$q^i = \frac{p_\theta^i + p_\alpha^i - t^i \sum_\alpha p_\alpha^i}{1-\left(1-t^l\right)\sum_\alpha p_\alpha^i}. \tag{1.48}$$

Benito-Garcia and Lopez-Ruiz (1992) derived the same equation (1.47) from a different approach. The step-by-step approach presented here is based on clear definitions and may help to better understand the parameters and terms related to the change of mineral proportions in the residual solid during incongruent melting.

Example. Assuming that this are no congruent minerals in the source, calculate q^i and x^i for the following reaction melting:

0.06 ol + 0.71 cpx + 0.23 gt = 0.52 melt +0.48 opx

(ol=olivine; cpx=clinopyroxene; gt=garnet; opx=orthopyroxene)
Based on definitions, we have

$$p_\alpha^{ol} = 0.06,\ p_\alpha^{cpx} = 0.71,$$

$$p_\alpha^{gt} = 0.06,\ t^l = 0.52,$$

$$t^{opx} = 0.48.$$

Since there are no congruent minerals we have $p_\theta^i = 0$, $\sum_\theta p_\theta^i = 0$ and $\sum_\alpha p_\alpha^i = 1$. For incongruent minerals in the reaction (ol, cpx, gt),

$$q^i = p_\alpha^i / t^l,$$

and for newly formed mineral (opx), we have

$$q^i = -t^i / t^l.$$

Thus, we have

$$q^{ol} = 0.06/0.52 = 0.12,$$

$$q^{cpx} = 0.71/0.52 = 1.37,$$

$$q^{gt} = 0.23/0.52 = 0.44,$$

$$q^{opx} = -0.48/0.52 = -0.92 .$$

The variations of the mineral proportions are

$$x^{ol} = \frac{x_0^{ol} - 0.12F}{1-F},$$

$$x^{cpx} = \frac{x_0^{cpx} - 1.37F}{1-F},$$

$$x^{gt} = \frac{x_0^{gt} - 0.44F}{1-F},$$

$$x^{opx} = \frac{x_0^{opx} + 0.92F}{1-F}.$$

1.4.1. Incongruent batch melting with constant K^i

For constant K^i, we have

$$K^i = K_0^i . \tag{1.49}$$

Substitution of (1.49) and (1.47) into (1.4) gives

$$D = \frac{1}{1-F}\left[D_0 - \frac{P_0 - \left(\sum_{\alpha} p_\alpha^i\right)\left(\sum_{\beta} t^i K_0^i\right)}{1-\left(1-t^l\right)\left(\sum_{\alpha} p_\alpha^i\right)} F \right] = \frac{D_0 - Q_0 F}{1-F}, \tag{1.50}$$

where

$$Q_0 = \sum q^i x_0^i = \frac{P_0 - \left(\sum_{\alpha} p_\alpha^i\right)\left(\sum_{\beta} t^i K_0^i\right)}{1-\left(1-t^l\right)\left(\sum_{\alpha} p_\alpha^i\right)} . \tag{1.51}$$

Substitution of (1.50) into (1.5) gives rise to

$$C_L = \frac{C_0}{D_0 + F(1 - Q_0)}. \tag{1.52}$$

Example. Find Q_0 for the melting reaction

0.06 ol + 0.71 cpx + 0.23 gt = 0.52 melt +0.48 opx.

Since there are no congruent minerals in the above melting reaction, we have $\sum_\alpha p_\alpha^i = 1$. On the basis of the melting reaction, we have

$$p_\alpha^{ol} = 0.06, \; p_\alpha^{cpx} = 0.71,$$

$$p_\alpha^{gt} = 0.06, \; t^l = 0.52,$$

$$t^{opx} = 0.48.$$

Substitution of these parameters into Eq. (1.51) gives

$$Q_0 = \frac{P_0 - \left(\sum_\beta t^i K_0^i\right)}{t^l} = \frac{P_0 - t^{opx} K_0^{opx}}{t^l}$$

$$= \frac{p^{ol} K^{ol} + p^{cpx} K^{cpx} + p^{gt} K^{gt} - t^{opx} K^{opx}}{0.52}$$

$$= \frac{0.06K^{ol} + 0.71K^{cpx} + 0.23K^{gt} - 0.48K^{opx}}{0.52}.$$

Alternatively, Q_0 can be obtained using q^i from the example from section 1.3.1:

$$Q_0 = \sum q^i x_0^i = 0.12K^{ol} + 1.37K^{cpx} + 0.44K^{gt} - 0.92K^{opx}.$$

Example. Find Q_0 for the melting reaction

0.27 qz + 0.43 bio + 0.30 sil = 0.72 cord +0.28 melt

(qz=quartz, bio=biotite, sil=sillimanite; cord=cordirite).

On the basis of the above melting reaction, we get

$$\sum_\alpha p_\alpha^i = 1, \; p_\alpha^{qz} = 0.27,$$

$$p_\alpha^{bio} = 0.43\,,\ p_\alpha^{sil} = 0.30\,,$$

$$t^l = 0.28\,,\ t^{cord} = 0.72\,.$$

Substitution of these parameters into Eq. (1.51) leads to

$$Q_0 = \frac{P_0 - \left(\sum_\beta t^i K_0^i\right)}{t^l} = \frac{P_0 - t^{cord} K_0^{cord}}{t^l}$$
$$= \frac{p^{qz} K^{qz} + p^{bio} K^{bio} + p^{sil} K^{sil} - t^{cord} K^{cord}}{0.52}$$
$$= \frac{0.27 K^{ol} + 0.43 K^{cpx} + 0.30 K^{gt} - 0.72 K^{opx}}{0.28}.$$

1.4.2. Incongruent batch melting with linear change of K^i

We may assume linear variations of distribution coefficients

$$K^i(F) = K_0^i + a^i F\,. \tag{1.53}$$

Combination of (1.53), (1.47) and (1.4) yields

$$\begin{aligned} D &= \Sigma x^i K^i \\ &= \Sigma \frac{x_0^i - F q^i}{1-F}\left(K_0^i + a^i F\right) \\ &= \frac{1}{1-F}\left[-\left(\Sigma a^i q^i\right) F^2 + \left(-Q_0 + \Sigma a^i x_0^i\right) F + D_0\right], \end{aligned} \tag{1.54}$$

where

$$D_0 = \sum K_0^i x_0^i \ \text{ and } \ Q_0 = \sum q^i x_0^i\,.$$

Substitution of Eq. (1.54) into Eq. (1.5) results in

$$C_L = \frac{C_0}{D_0 + \left(1 - Q_0 + \sum a^i x_0^i\right) F - \left(\sum a^i q^i\right) F^2}\,. \tag{1.55}$$

Note that if all $a^i = 0$, then Eq. (1.55) reduces to Eq. (1.52).

1.4.3. Incongruent batch melting with linear change of K^i and linear change of q^i

If q^i is also a function of F, then Eq. (1.46) for mass conservation of phase i should be replaced by

$$x^i(1-F) = x_0^i - \int_0^F q^i dF . \tag{1.56}$$

We may assume linear variations in q^i during incongruent melting

$$q^i = q_0^i + b^i F , \tag{1.57}$$

where $b^i = \left(p^i(F_{\max}) - p_0^i\right)/F_{\max}$, $F_{\max}$ is the maximum degree of melting when one of the phases melt completely, and $q^i(F_{\max})$ is the net fractional contribution to the melt when the degree of partial melting is $F_{\max}$. Substitution of (1.57) into (1.56) yields

$$x^i = \frac{x_0^i - Fq_0^i - 0.5b^i F^2}{1-F} . \tag{1.58}$$

Combination of (1.58), (1.53) and (1.4) gives

$$D = \frac{\left[B_0F^3 + B_1F^2 + B_2F + D_0\right]}{1-F}, \tag{1.59}$$

where

$$B_0 = -\left(\sum 0.5a^i b^i\right), \tag{1.60}$$

$$B_1 = -\sum a^i q_0^i - \sum 0.5b^i K_0^i , \tag{1.61}$$

$$B_2 = \sum a^i x_0^i - \sum q_0^i K_0^i = \sum a^i x_0^i - Q_0, \tag{1.62}$$

$$D_0 = \sum K_0^i x_0^i . \tag{1.63}$$

Substitution of Eq. (1.59) into Eq. (1.5) results in

$$C_L = \frac{C_0}{D_0 + (1+B_2)F + B_1F^2 + B_0F^3} . \tag{1.64}$$

Note that if all $b^i = 0$, then Eq. (1.64) reduces to Eq. (1.55). If all $a^i = 0$ and all $b^i = 0$, then Eq. (1.64) collapses to Eq. (1.52).

1.5. Summary

Table 1.2. Summary of variations of mineral proportions and bulk partition coefficients during modal, eutectic and incongruent melting.

Melting modes	Mineral proportions	Bulk partition coefficients
Modal melting	$x^i = x_0^i$	$D = \sum x_0^i K^i$
Eutectic Melting	$x^i(F) = \frac{x_0^i - Fp^i}{1-F}$	$D = \sum \frac{x_0^i - Fp^i}{1-F} K^i$
Incongruent melting	$x^i(F) = \frac{x_0^i - Fq^i}{1-F}$	$D = \sum \frac{x_0^i - Fq^i}{1-F} K^i$

Table 1.3. Relationships among different batch melting (BM) models in this chapter.

	Constant K^i	Linear K^i	Linear K^i and q^i
Incongruent Batch Melting	Incongruent BM w/ constant K^i	Incongruent BM w/ linear K^i	Incongruent BM w/ linear K^i and linear q^i
	$q^i = p^i \downarrow$	$q^i = p^i \downarrow$	$q^i = p^i \downarrow$
Eutectic Batch Melting	Eutectic BM w/ constant	Eutectic BM w/ linear K^i	Eutectic BM w/ linear K^i and linear q^i
	$x_i = x_i^0 \downarrow$	$x_i = x_i^0 \downarrow$	$x_i = x_i^0 \downarrow$
Modal Batch Melting	Modal BM w/ constant	Modal BM w/ linear K^i	Modal BM w/ linear K^i and linear q^i

The important equations for the various batch melting models are summarized as follows:

1) Modal batch melting

$$C_L = \frac{C_0}{D_0 + F(1 - D_0)}.$$

2) Eutectic batch melting

Constant K^i and constant p^i:

$$C_L = \frac{C_0}{D_0 + F(1 - P)}.$$

Linear K^i and constant p^i:

$$C_L = \frac{C_0}{D_0 + \left(1 - P_0 + \sum a^i x_0^i\right) F - \left(\sum a^i p^i\right) F^2}.$$

Linear K^i and linear p^i:

$$C_L = \frac{C_0}{D_0 + \left(1 + A_2\right) F + A_1 F^2 + A_0 F^3},$$

where

$$A_0 = -\left(\sum 0.5 a^i b^i\right),$$

$$A_1 = -\sum a^i p_0^i - \sum 0.5 b^i K_0^i,$$

$$A_2 = \sum a^i x_0^i - \sum p_0^i K_0^i = \sum a^i x_0^i - P_0.$$

3) Incongruent batch melting

Constant K^i and constant q^i:

$$C_L = \frac{C_0}{D_0 + F(1-Q_0)}.$$

Linear K^i and constant q^i:

$$C_L = \frac{C_0}{D_0 + \left(1 - Q_0 + \sum a^i x_0^i\right)F - \left(\sum a^i q^i\right)F^2}.$$

Linear K^i and linear q^i:

$$C_L = \frac{C_0}{D_0 + (1 + B_2)F + B_1 F^2 + B_0 F^3},$$

where

$$B_0 = -\left(\sum 0.5 a^i b^i\right),$$

$$B_1 = -\sum a^i q_0^i - \sum 0.5 b^i K_0^i,$$

$$B_2 = \sum a^i x_0^i - \sum q_0^i K_0^i = \sum a^i x_0^i - Q_0.$$

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