Preface

This text forms what is often referred to as "a first course in complex analysis". It is a slight enhancement of lecture notes first presented to undergraduate students in the Mathematics Department of Bedford College, University of London, as part of the Mathematics BSc. degree, and then given for many years in the Mathematics Department of King's College, London. During this time they have been continually revised, reorganized and rewritten. The aim was to provide a rigorous and largely self-contained but extremely gentle introduction to the basics of complex analysis.

The audience for the course comprised not only single subject mathematics BSc. and MSci. students but also a number of final year joint honours students as well as postgraduate students who missed out on the subject in their undergraduate programme.

There are a number of core topics (such as Cauchy's theorem, the Taylor and Laurent series, singularities and the residue theorem) which simply must be offered to any student of complex analysis. However, quite a bit of preparation is required, so these important results unavoidably tend to appear in rather rapid succession towards the end the course. This leaves very little room for extra topics, especially if they are particularly complicated or involve a lot of additional machinery. The presentation here is for the benefit of the student audience. There has been no quest for ultimate generality nor economy of delivery.

Nowadays, it seems that many students do not get to see an account of metric spaces, so this aspect of complex analysis has been presented in quite some detail (in Chapter 3). It is then but a small step for the student wishing to go on to study metric spaces in general. The exponential and trigonometric functions are defined via their power series expansions in Chapter 5, so a certain amount of manœuvring is required to extract those properties familiar from calculus—for example, the appearance of the number π is carefully explained. Those for whom this is familiar territory can quickly press on.

Most of the core results are contained in Chapters 8–12. The next two chapters, covering the maximum modulus principle and Möbius transformations have been moved around a bit over the years. For example, the maximum modulus principle (Chapter 13) could be discussed anytime after having dealt with Cauchy's integral formulae (Chapter 8). The treatment of Möbius transformations (Chapter 14) is essentially a stand-alone topic so could fit in almost anywhere. It might well be read after Chapter 8 so as to provide a little variety before embarking on the study of the Laurent expansion in Chapter 9.

It has to be admitted that the final section of Chapter 13 (on Hadamard's Theorem), possibly Chapter 15 (on harmonic functions) and Chapter 16 (on local properties of analytic functions) could be considered a bit of a luxury. In practice, they were all usually squeezed out because of lack of time. Most of the rest of the material in these notes just about fits into a one semester course.

The majority of students embarking on this subject will have studied calculus and will usually have also been exposed to some real analysis. Nevertheless, experience has shown that the odd reminder does not go amiss and so an appendix containing some pertinent facts from real analysis has been included. These are all consequences of the completeness property of $\mathbb{R}$ (so tend not to be very carefully covered in calculus courses—or else are deemed obvious).

A number of text books were consulted during the preparation of these notes and these are listed in the bibliography. No claim is made here regarding originality. As an undergraduate student in the Mathematics Department of Imperial College, it was my privilege to be taught analysis by M. C. Austin and Professor Ch. Pommerenke. Their lectures could only be described as both a joy and an inspiration. It is a pleasure to acknowledge my indebtedness to them both.

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