Contents

Preface v

1. Mappings in Metric and Normed Spaces 1

1.1 Topological Spaces 1
1.1.1 Topology 1
1.1.2 Neighborhoods 1
1.1.3 Examples of topologies 2
1.1.4 Interiors and closures. Limit points 3
1.1.5 Dense subsets and separable spaces 4
1.1.6 Induced topology. Subspaces 4
1.1.7 Continuous mappings 5
1.1.8 Compactness 6
1.1.9 Ordered sets 7
1.1.10 Topological vector spaces 7
1.2 Metric Spaces 8
1.2.1 Metrics and pseudometrics (semimetrics) 8
1.2.2 Examples 11
1.2.3 Completeness 12
1.2.4 Compactness and boundedness 12
1.3 Normed and Banach Spaces 12
1.3.1 Norms on a vector space 12
1.3.2 Examples 13
1.4 Hilbert Spaces 14
1.4.1 Scalar product 14
1.4.2 Examples 15
1.5 Locally Convex Spaces 17

1.5.1 Convex sets and convex hulls 17
1.5.2 Extreme points 18
1.5.3 Examples 19
1.5.4 The topology induced by seminorms 19
1.5.5 The Minkowski functional 20
1.5.6 Locally convex spaces and seminorms 21
1.6 Linear and Multilinear Mappings in Banach Spaces 22
1.6.1 Linear operators 22
1.6.2 Examples 24
1.6.3 The space of bounded linear operators 25
1.6.4 Multilinear mappings and polynomials 25
1.6.5 Banach algebra of linear operators 27
1.6.6 Spectra and resolvents of linear operators 29
1.6.7 Examples 32
1.7 Duality in Normed Spaces 33
1.7.1 Dual spaces 33
1.7.2 Examples 33
1.7.3 Weak topology and reflexivity 33
1.7.4 The weak and weak* topologies 35
1.8 The Hahn–Banach Theorem 36
1.8.1 The extension theorem 36
1.8.2 The completion of a normed space 38
1.8.3 Geometric Hahn–Banach separation theorems 38
1.9 Elements of Ergodic Theory 40
1.9.1 Mean ergodic theorem 40
1.9.2 Uniform ergodic theorems in Banach spaces 42
1.10 Lipschitzian and Nonexpansive Mappings in Metric Spaces . 44
1.10.1 Lipschitzian and contraction mappings 44
1.10.2 Nonexpansive mappings 45
1.10.3 Uniformly Lipschitzian mappings 45
1.10.4 Firmly nonexpansive mappings 46
1.10.5 Monotone and accretive mappings 47

2. Differentiable and Holomorphic Mappings in Banach Spaces 51

2.1 Differentiable Mappings. Fréchet Derivatives 51
2.1.1 Examples 53
2.2 Holomorphic Mappings 54
2.2.1 The Cauchy integral formula 55
2.2.2 Power series representation 56

2.2.3 The maximum modulus theorem 57
2.3 Topologies in $\mathrm{Hol}(\mathcal{D}, Y)$ 60
2.3.1 T-topology and compact open topology on $\mathrm{Hol}(\mathcal{D}, Y)$ 60
2.3.2 Montel's theorem 61
2.3.3 Vitali's theorem 62
2.4 Elements of Functional Analytic Calculus 63
2.4.1 Symbolic calculus on Banach algebras 63
2.4.2 The spectral mapping theorem 65
2.4.3 Some $*$-algebras 66
2.4.4 l-analytic functions on unital J^*-algebras 68
2.5 The Schwarz Lemma 69
2.5.1 The classical Schwarz Lemma and Cartan's uniqueness theorem 69
2.6 Automorphisms 72
2.6.1 The unit disk 72
2.6.2 The polydisk in $\mathbb{C}^n$ 74
2.6.3 The Euclidean ball in $\mathbb{C}^n$ and the Hilbert ball 74
2.6.4 Unital J^*-algebras 76
2.6.5 The Schwarz–Pick lemma 76

3. Hyperbolic Metrics on Domains in Complex Banach Spaces 81
3.1 The Poincaré Metric on the Unit Disk 81
3.2 The Infinitesimal Poincaré Metric and Geodesics 86
3.3 The Poincaré Metric on the Hilbert Ball and its Powers . . 88
3.4 The Carathéodory and Kobayashi Pseudometrics 89
3.4.1 The Carathéodory pseudometric 89
3.4.2 The Kobayashi pseudometric 91
3.5 Infinitesimal Finsler Pseudometrics 93
3.5.1 Examples 95
3.6 Schwarz–Pick Systems of Pseudometrics 97
3.7 Bounded Convex Domains and Metric Domains in Banach Spaces 101

4. Some Fixed Point Principles 107
4.1 The Banach Principle 107
4.2 The Theorems of Brouwer and Schauder 110
4.3 Holomorphic Fixed Point Theorems 111
4.4 Fixed Points in the Hilbert Ball 115

4.5 Fixed Points in Finite Powers of the Hilbert Ball 116

5. The Denjoy–Wolff Fixed Point Theory 119

5.1 The One-Dimensional Case 119
5.1.1 Iterates of holomorphic self-mappings of Δ with an interior fixed point 119
5.1.2 Iterates of holomorphic self-mappings of Δ with no interior fixed point 121
5.2 The Unit Hilbert Ball 127
5.3 Convex Domains in $\mathbb{C}^n$ 135
5.4 Domains in Banach Space. 138
5.5 Holomorphic Retracts and the Structure of the Fixed Point Sets. 144

6. Generation Theory for One–Parameter Semigroups 157

6.1 Continuous and Discrete One-Parameter Semigroups on Metric Spaces 157
6.1.1 Discrete and continuous flows on a domain 157
6.1.2 Examples 159
6.2 Linear semigroups 167
6.3 Generated Semigroups of Nonexpansive and Holomorphic Mappings 175
6.4 The Cauchy Problem and the Product Formula 183
6.5 Nonlinear Resolvents, the Range Condition and Exponential Formulas 190

7. Flow–Invariance Conditions 199

7.1 Boundary Flow Invariance Conditions 199
7.2 Numerical Range of Holomorphic Mappings 202
7.3 Interior Flow Invariance Conditions 207
7.4 Semi-Complete and Complete Vector Fields 213

8. Stationary Points of Continuous Semigroups 219

8.1 Generalities 219
8.2 Generated Semigroups 226
8.3 The Resolvent Method 228
8.4 Null Point Free Generators 232

8.5 The Structure of Null Point Sets of Holomorphic Generators. Retractions 237
8.6 A Stabilization Phenomenon 244
8.7 Local and Spectral Characteristics of Stationary Points . . 248
8.7.1 Cartan's uniqueness theorem 248
8.7.2 Harris' spectrum of a semi-complete vector field . . . 249

9. Asymptotic Behavior of Continuous Flows 253

9.1 Strongly Semi-Complete Vector Fields in Banach Spaces . . 253
9.2 Asymptotic Behavior of Flows of ρ-Nonexpansive Mappings on the Hilbert Ball 262
9.3 Flows of Holomorphic Mappings on the Hilbert Ball 272
9.3.1 Interior stationary point 273
9.3.2 Boundary sink point. Continuous version of the Julia–Wolff–Carathéodory theorem 280
9.4 Admissible Lower and Upper Bounds and Rates of Convergence 286

10. Geometry of Domains in Banach Spaces 297

10.1 Biholomorphic Mappings in Banach Spaces and Generators on Biholomorphically Equivalent Domains 297
10.2 Starlike, Convex, and Spirallike Mappings 300
10.2.1 Starlike functions on the unit disk 301
10.2.2 Convex and close-to-convex functions on the unit disk 303
10.2.3 Spirallike functions on the unit disk 304
10.3 Higher-Dimensional Extensions and the Dynamical Approach 304
10.4 Distortion Theorems for Starlike Mappings on the Unit Ball 316
10.5 Differential Equations for Starlike and Spirallike Mappings in $\mathcal{H} = \mathbb{C}^n$ 324

Bibliography 339

Index 351