

Preface

The outline of this volume is based roughly upon a series of lectures given to the Group Theory Seminar in the Department of Mathematics at the University of Illinois, Urbana-Champaign, in the fall of 2002. The text's content is drawn from the robust literature on questions related to the Isomorphism Problem for Coxeter groups. The material contained in that literature has been augmented with a sizable amount of preliminary material in order to make the text somewhat self-contained. With this supplementary material, the text should be accessible to any graduate student (or mature undergraduate student) who has taken courses in linear algebra and group theory: (For instance, basic knowledge of group theory, including group presentations and group actions is assumed.) Following is an outline of the text.

After a brief sketch of the historical development of the study of Coxeter groups, the first chapter provides an introduction to the general theory of these groups. A number of exercises are included in order to allow the student who is less familiar with Coxeter groups the opportunity to explore their nature. Chapter 1 should give the reader the background needed to approach the topics covered later in the text. Moreover, the first chapter highlights the interplay between the combinatorial and geometric characterizations of Coxeter groups in order to demonstrate to the reader the usefulness of both of these points of view.

The second chapter builds upon the first by surveying specific properties of Coxeter groups of importance later in the text. It is in this chapter that we compute the normalizers and centralizers of important subgroups of a given Coxeter group. Here we also discuss decompositions of Coxeter groups as free products with amalgamation, and we describe a few homological

properties of Coxeter groups. A brief survey of the Word and Conjugacy Problems is provided as well. Much of this information will be used in later chapters.

The third chapter begins by outlining various characterizations of uniqueness in Coxeter groups, including the concept of rigidity. The questions with which the remainder of the text concerns itself are then stated. This chapter contains few proofs, as here we seek merely to provide an overview of the results regarding rigidity while indicating the ways in which they are related to one another.

The next four chapters fill in the gaps in rigor contained in Chapter 3. Reasonably complete proofs of primary results are provided (the reader is often asked to supply details in the exercises). Roughly speaking, we address the primary results in such a manner as to separate those whose proofs require arguments of a geometric flavor from those whose proofs are more combinatorial. However, as the reader will notice, it is often difficult to separate results in this fashion.

In the final chapter we address questions related to the Isomorphism Problem. Here, for instance, we investigate the structure of $\text{Aut}(w)$ for a given Coxeter group W . Chapter 7 also considers the concept of rigidity in the setting of Artin groups.

This volume is in no way meant to be a comprehensive treatment of Coxeter groups. The reader should be aware that these groups are of considerable importance in a number of fields not addressed in this book, including low-dimensional topology, Lie theory, representation theory, and algebraic geometry.

As of the writing of this text, the Isomorphism Problem for Coxeter groups (the “Holy Grail” of the study of rigidity) remains unsolved. However, it is possible, if not likely, that a solution to this problem will be discovered quite soon. Such a solution may or may not make use of methods similar to those examined in this volume. In any case, a number of the partial results that we consider here may be rendered obsolete. Why, then, consider this text?

It is my hope that the reader will take from the text not only an understanding of the questions involved in approaching the Isomorphism Problem, but also an appreciation for the various methods with which such questions are addressed. Indeed, these methods highlight the structure and beauty of the groups themselves, and provide direction for investigation

into properties not considered here. It is for this reason, if for no other, that so many different viewpoints are adopted in the course of our study. Each of these viewpoints has its own merits, and each leads to methods and machinery that are applicable in the solution of many other problems in the theory of Coxeter groups.

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