

Contents

<i>Preface</i>	vii
1. Infinite Lattice Systems	1
1.1 Equations of motion	1
1.2 The Cauchy problem	7
1.3 Harmonic lattices	10
1.4 Chains of coupled nonlinear oscillators	16
1.5 Comments and open problems	24
2. Time Periodic Oscillations	27
2.1 Setting of problem	27
2.2 Positive definite case	34
2.3 Indefinite case	42
2.3.1 Main result	42
2.3.2 Periodic approximations	46
2.3.3 Proof of main result	54
2.4 Additional results	56
2.4.1 Degenerate case	56
2.4.2 Constrained minimization	58
2.4.3 Multibumps	59
2.4.4 Lattices without spatial periodicity	61
2.4.5 Finite lattices	62
2.5 Chains of oscillators	64
2.6 Comments and open problems	70
3. Travelling Waves: Waves with Prescribed Speed	75

3.1	Statement of problem	75
3.2	Periodic waves	78
3.2.1	Variational setting	78
3.2.2	Monotone waves	81
3.2.3	Nonmonotone and subsonic waves	85
3.3	Solitary waves	89
3.3.1	Variational statement of the problem	89
3.3.2	From periodic waves to solitary ones	93
3.3.3	Global structure of periodic waves	101
3.3.4	Examples	104
3.4	Ground waves: existence and convergence	105
3.4.1	Ground waves: periodic case	105
3.4.2	Solitary ground waves	109
3.4.3	Monotonicity	112
3.5	Near sonic waves	114
3.5.1	Amplitude estimate	114
3.5.2	Nonglobally defined potentials	117
4.	Travelling Waves: Further Results	121
4.1	Solitary waves and constrained minimization	121
4.1.1	Statement of problem	121
4.1.2	The minimization problem: technical results	123
4.1.3	The minimization problem: existence	133
4.1.4	Proof of main result	140
4.1.5	Lennard-Jones type potentials	143
4.2	Other types of travelling waves	146
4.2.1	Waves with periodic profile functions	146
4.2.2	Solitary waves whose profiles vanish at infinity	148
4.3	Yet another constrained minimization problem	150
4.4	Remark on FPU β -model	152
4.5	Exponential decay	154
4.6	Travelling waves in chains of oscillators	160
4.7	Comments and open problems	163
	Appendix A Functional Spaces	167
	A.1 Spaces of sequences	167
	A.2 Spaces of functions on real line	168
	Appendix B Concentration Compactness	173

Appendix C	Critical Point Theory	177
C.1	Differentiable functionals	177
C.2	Mountain pass theorem	178
C.3	Linking theorems	179
Appendix D	Difference Calculus	183
	<i>Bibliography</i>	185
	<i>Index</i>	193