Chapter 1

Introduction

Black holes are among the most fascinating predictions of Einstein's theory of general relativity. They are an exotic, but natural outcome of the very basic feature of the gravitational interaction. The universality of gravity, nicely expressed by the equivalence principle, allows to produce an accumulative effect that can result in a very strong gravitational field. Indeed, it can be so strong that it cannot be counterbalanced by other forces and continues to grow up until extreme situations. Since gravity affects the spacetime geometry, the gravitational field produced by matter could become so strong as to substantially modify the ordinary causal structure of spacetime, and produce a region where even light is trapped and no particle can escape to infinity.

Quantum mechanics enters immediately into this game and its first consequence is to prevent, partly, this extreme situation. Chandrasekhar showed that the quantum degeneracy pressure of electrons can balance the gravitational force and avoid a complete implosion. However, this requires a limiting mass for the star of about 1.4 times the solar mass. Above this limit the full gravitational collapse can be prevented only by the degeneracy pressure of neutrons, but this requires again an upper mass limit (2–3 solar masses). Therefore for sufficiently massive stars, that do not throw away enough matter or radiation to reach the neutron star limit, complete collapse is inevitable and a black hole forms.

Nevertheless, quantum mechanics seems to continue conspiring against the black hole, but in a different way. In a remarkable discovery Hawking showed how this takes place. Black holes are not as "black" as general relativity predicts, but rather radiate thermally all types of existing quanta, although for kinematical reasons light particles dominate the emission. This is a very small effect, at least initially, for black holes created from gravitational collapse. Hawking's discovery, which can be rederived from different perspectives, puts forward an intriguing and close relation between black holes and thermodynamics. This astonishing result is obtained using the approximation that the spacetime background is kept fixed at all times. This is an accurate approximation for the first period of the evaporation, where the effect is small, and, likely, a reasonable approximation until the black hole reaches Planck-size. Eventually, by extrapolation of the results, the black hole will disappear. The conspiracy of quantum mechanics is such that it turns, as a "boomerang" effect, against itself. Indeed, the resulting physical picture, as it was pointed out by Hawking himself almost thirty years ago, seems not to be compatible with the principles of quantum mechanics. The type of radiation emitted does not allow the recovery of the information about the star from which the black hole was created. Therefore, with the disappearance of the black hole this information will be lost forever. But this is forbidden by the basic principles of quantum mechanics itself.

However, there is no definitive picture of the full evaporation process and the reason comes out immediately. The fixed background approximation ignores the effects of the radiation on the spacetime geometry, in other words, the backreaction effects. They play an important role, even before reaching the Planck scale, when a still unknown quantum theory of gravity is expected to dominate the process. They need to be taken into account to have a detailed view of the evaporation process, and indeed they could serve as an inspiration to attack the deep problems or paradoxes of black hole evaporation mentioned above. The hope of many researchers is that, at the end, quantum mechanics will keep conspiring in such a way that a full quantum gravity approach will modify Hawking's original picture and allow information retrieval. Most remarkably, Hawking himself now seems to be converging towards this belief after almost thirty years of skepticism.

In general it is hard to try to go beyond the fixed background approximation to include the backreaction, even at the semiclassical level. This requires to solve the so called semiclassical Einstein equations

$$G_{\mu\nu} = 8\pi\langle\Psi|T_{\mu\nu}(g_{\alpha\beta})|\Psi\rangle \ . \tag{1.1}$$

This is not at all a purely technical problem. First of all one needs to have an explicit expression for the expectation values $\langle\Psi|T_{\mu\nu}(g_{\alpha\beta})|\Psi\rangle$ for a large family of metrics (including those that could be potentially the solution of the semiclassical equations). Moreover, these quantities also depend on the quantum state of the matter $|\Psi\rangle$, and this is a rather non-trivial issue.

To properly model the process of black hole evaporation one should bypass these difficulties somewhat. This is a remarkable open problem which should be addressed, in the appropriate limit, by any theory containing general relativity and quantum mechanics.

The organization of the material follows the above brief historical "tour". In Chapter 2 we start by briefly overviewing the Oppenheimer-Snyder model and the main features of the gravitational collapse. After this we introduce the basic ingredients of stationary (charged and rotating) black holes, using the simplest one, i.e., the Schwarzschild solution. To make the presentation accessible for a wide community of scientists we have based our arguments, as much as possible, on the equivalence principle to derive the basic results. The Kruskal coordinates, usually introduced for global purposes, are motivated here on a local basis in terms of locally inertial coordinates, where the intuition of readers more familiar with flat spacetime physics is more solid. Nevertheless the global aspects of the solutions are, of course, very important and we also pay special attention to this issue. In addition to the most popular notion of "event horizon" we also introduce the concept of "apparent horizon", which is of special relevance in the analysis of time dependent settings such as the evaporation process of Chapter 6. We also present the intriguing formal analogy between the "laws of black hole mechanics" and the laws of thermodynamics. We point out the loophole of the analogy, which will be filled in as a result of Hawking's discovery (to be presented in Chapter 3). Finally we also introduce black hole solutions in theories different from pure general relativity, motivated by the fact that they will be the inspiration of the model proposed by Callan, Giddings, Harvey and Strominger, the main theme of Chapter 6.

The whole Chapter 3 is devoted to the Hawking effect. We shall follow the original derivation of Hawking, together with the works of Parker and Wald, to explicitly show that the radiation emitted by a black hole at late times is exactly described by a thermal density matrix (modulated by a "grey-body" factor). We divide the derivation in two steps aiming to show in a clear way the skeleton of the argument and separate the technicalities from the main physical ideas. The simplest possible model, involving the matching of Minkowski and Schwarzschild spacetimes, contains all the ingredients that produce the Hawking radiation: the existence of an event horizon in a non-stationary spacetime. The intention is to introduce the Hawking radiation in an elementary way and make it more accessible to readers with a basic background on general relativity. The second step is to show that the late time thermal radiation obtained in this simplified spacetime, mimicking a gravitational collapse, is insensitive to the details of a realistic collapse. Apart from this, the derivation is quite standard. We based it on the properties of the late time Bogolubov transformations and the features of wave propagation in the Schwarzschild geometry. We finish the chapter presenting the physical implications of the Hawking effect. We shall discuss in detail the challenges that black hole evaporation poses to the interphase of quantum mechanics and general relativity. The possible breakdown of quantum predictability is the cornerstone of this more conceptual discussion. It will serve to warn the reader that not everything in this subject is well understood.

In Chapter 4 we approach the Hawking effect from a different perspective, aiming to get a better understanding. Since the basic feature is the existence of a horizon we simplify the Schwarzschild geometry working with its "near-horizon approximation" Rindler geometry, which is nothing else but a wedge of Minkowski space. We want to remark that it is quite common in the literature to discuss Rindler space and the Unruh effect before considering its curved spacetime "analog". However, we prefer to present things the other way around. The Unruh effect is even more shocking than the Hawking effect for readers that have learned "quantum field theory" with too much emphasis on "Poincaré symmetry" (see at this respect [Wald (1994)]). For this reason we rederive Hawking radiation from the near-horizon Rindler geometry and, only as a by-product, the Unruh effect. This result finds justification from the equivalence principle. All this discussion allows us to introduce into the game, in a natural way, one of the strongest spacetime symmetries in physics: the two-dimensional conformal symmetry. The theory of conformal fields is usually applied to analyse second-order phase transitions in condensed matter systems [Di Francesco *et al.* (1997)] and it is also widely used in the formulation of string theory [Polchinski (1998)]. We shall show how it also plays an important role in the Hawking effect. The thermal character of the emitted radiation can be nicely reobtained in terms of the conformal properties of the correlation functions of the effective matter theory around the horizon. This will offer a new perspective to grasp the deep physical meaning of the Hawking effect.

In Chapter 5 we approach the problem of determining an expression for the expectation value of the stress tensor for the matter fields in a curved background. This is the first obstacle one faces before attacking the backreaction problem. This is the fundamental problem of "quantum field theory in curved spacetime" and it is indeed very hard. No exact analytical expression is known even for the fixed Schwarzschild spacetime. Only for conformally flat geometries and for conformal fields one has a solution to the problem. We are far away from having such an expression in a generic four-dimensional spacetime. Reduction of the gravity-matter system under spherical symmetry leads to an effective two-dimensional theory that can serve as the starting point for the analysis. The advantage of doing so is that in two dimensions every metric is conformally flat, and therefore one expects to find an expression for the expectation value of the stress tensor. The near-horizon approximation, together with the ensuing conformal symmetry, allows to find an exact solution to this problem. This result is usually presented as a consequence of the fact that the trace anomaly determines univocally all the components of the stress tensor. An understanding of this, at least for a non-expert reader, will require a derivation of the trace anomaly itself. We want to avoid entering into the technicalities of regularization in curved spacetime and for this reason we present an unconventional derivation of the expectation values of the stress tensor, including the trace anomaly itself. Our derivation is based on the use of locally inertial coordinates and of the transformation law of the normal ordered stress tensor obtained in Chapter 4. In this way a non expert reader can easily follow the derivation. This approach also allows to determine an expression for the stress tensor when the spherically symmetric reduction is not restricted to the near-horizon geometry. We consider in detail the problem of selecting the appropriate quantum states relevant for quantum black hole physics. We stress that this is a highly non-trivial problem, especially when addressing the backreaction problem for the case of evaporating black holes. The last part of the chapter is devoted to this issue.

The first model which bypasses all these difficulties was proposed twelve years ago in [Callan *et al.* (1992)] and describes the near-horizon properties of near-extremal stringy black holes. Starting from it, the first analytic description of the process of black hole formation and evaporation was given in [Russo *et al.* (1992b)]. This is the central theme of Chapter 6. We also study the near-horizon dynamics of near-extremal Reissner–Nordström black holes. Also in this case the semiclassical equations are solvable, despite the fact that the selection of the appropriate quantum state to describe the evaporation is more involved. On the basis of these exact solutions we discuss the implications of this approach for what concerns the information loss paradox, but also its limitations.

Notation: throughout this book we shall follow the conventions for the metric and the curvature used in [Misner *et al.* (1973)]. The metric signature is $(- + ++)$ and the definition of the curvature tensors are: $R^{\mu}{}_{\nu\alpha\beta} = \partial_\alpha \Gamma^{\mu}_{\nu\beta} - \cdots$, $R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$. In most of the book we use geometrized units $G = 1 = c$. However, to emphasize the quantum aspects we maintain explicitly Planck's constant ($\hbar$) in the formulae.