

Preface

This set of lecture notes presents a new approach to the representation theory of the symmetric group — more precisely: to the character theory of the symmetric group over a field of characteristic zero. Knowledge of the classical theory is unnecessary — perhaps, even a hindrance to the understanding of this new theory which is in many ways more immediate, more efficient, more transparent, and more elementary due to the new tools that lie behind the word “noncommutative”.

The ordinary character theory of the symmetric group is a commutative theory based on the commutative algebra of class functions or, equivalently, the algebra of symmetric functions. The monographs of James and Kerber [JK81] and of Macdonald [Mac95] are general references on the subject.

The noncommutative character theory is based on a noncommutative algebra which maps onto the algebra of class functions. The underlying idea is to transfer problems to the noncommutative superstructure, and to solve them in that setting.

Louis Solomon was the first to introduce this idea onto the character theory of the symmetric group, in a seminal article of 1976. Since then, Armin Jöllenbeck and Christophe Reutenauer have been among the main architects of the theory, as well as the members of the research group in Paris: Gérard Duchamp, Israel Gelfand, Florent Hivert, Daniel Krob, Alain Lascoux, Bernard Leclerc, Jean-Yves Thibon, to name a few. Our presentation of the material grew out of several lecture courses held in Kiel during the past years. It is self-contained and suitable for undergraduate level.

The main text is subdivided into three parts. Basic properties of the algebra of class functions are rederived in Part I. Part II contains the construction of the noncommutative superstructure and plays the key role here.

Various applications are then given in Part III.

We have included three appendices to supplement the main text, in rather different ways. Appendix A contains a short and self-contained dispatch of the character theory of finite groups in general, to the extent needed here. Appendix B contains a simple proof of Solomon's theorem. In Appendix C, a new approach is presented, to the Robinson-Schensted correspondence and related results of Knuth and Schützenberger which are at the combinatorial core of noncommutative character theory.

The reader is referred to the introduction for much more details on the ideas of the noncommutative theory and the thread of this manuscript.

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