Chapter 1

Introduction

This Chapter is a short technical introduction to microstructured optical fibres. Our aim is to introduce the notation and the fundamental concepts concerning optical fibres that will be encountered throughout the book. First, we review the principles and main properties of conventional optical fibres. Then we give a short overview of photonic crystals and light guidance in the defects that can be incorporated within a host lattice. This leads us to introduce the principles of guidance in photonic crystal fibres and more generally in microstructured optical fibres (MOFs). We also mention some of their advantages and applications. To conclude, we come back to a crucial issue: the nature of mode propagation in MOFs. The finite extent of the cladding structure of MOFs implies that all of the so-called guided modes are in fact leaky.

1.1 Conventional Optical Fibres

1.1.1 *Guidance mechanism*

Conventional optical fibres [SL83; Agr01] rely on total internal reflection to guide light. The simplest optical fibre – the step index fibre – consists of a dielectric core with refractive index n_{CO} (the refractive index is the square root of the relative permittivity) surrounded by another dielectric (called *cladding*) with refractive index n_{CL}. Using a ray approach and the Snell-Descartes law, it is easy to see that if $n_{\text{CO}} > n_{\text{CL}}$, light propagating in the core reaching the core/cladding interface is totally reflected back into the core as soon as the angle between the direction of propagation and the core/cladding interface is small enough.

More rigorously, we consider the infinitely long step index fibre with circular cross-section (radius ρ) depicted in Fig. 1.1. We suppose that the light propagating along the axis of the fibre (z-axis) has free-space wavelength λ and wavenumber k_0 ($k_0 = 2\pi/\lambda$). The studied system is invariant under any translation in the z-direction, which implies that the electromagnetic fields associated with an individual mode can have a dependence of $\exp(i\beta z)$ along z. β is called the *propagation constant.*[1] Therefore β is the common z-component of the wave vectors in the core and in the cladding. Since the norms of the wave vectors in the core and in the cladding are $n_{\text{CO}}k_0$ and $n_{\text{CL}}k_0$ respectively, β must be less than or equal to $n_{\text{CO}}k_0$ to propagate in the core and less or equal to $n_{\text{CL}}k_0$ to propagate in the cladding. If $n_{\text{CL}} < \beta/k_0 < n_{\text{CO}}$, light can propagate in the core, but not in the cladding: the light is trapped in the core. We will sometimes also use the *perpendicular wavenumber* $k_\perp$, defined by

$$k_\perp^2 + \beta^2 = n^2 k_0^2, \tag{1.1}$$

where n is the local refractive index (equal to n_{CO} or to n_{CL}). Since β is real in the case of conventional optical fibres with $n_{\text{CO}} > n_{\text{CL}}$, in the region where $\beta < nk_0$ we simply have $k_\perp = \sqrt{n^2k_0^2 - \beta^2}$ and $k_\perp = i\sqrt{\beta^2 - n^2k_0^2}$ elsewhere.

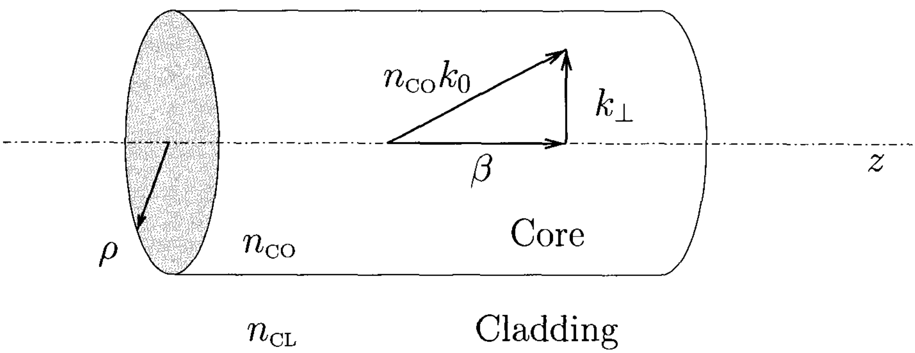

Fig. 1.1 Conventional step index fibre.

[1] Actually, a more general z dependency can be assumed but this is beyond the scope of this introduction.

1.1.2 *Fibre modes*

Considerations relating to total internal reflection or, equivalently the possible propagation constants in the core and the cladding give a necessary but not sufficient condition on β for light to be guided. Indeed, we have seen that the variation that the fields can have along z is given by the phase factor $\exp(i\beta z)$. Between two arbitrary values z_1 and z_2 of z, the fields propagate and undergo reflection at the core/cladding interface, but the transverse field distribution at $z = z_1$ and $z = z_2$ must only differ by the phase factor $\exp[i\beta(z_2 - z_1)]$. This defines a resonance condition, and only a discrete set of transverse field distributions and associated β values fulfill this condition. Mathematically speaking, the values β and their associated transverse field distributions are eigenvalues and associated eigenfunctions of the propagation equation [SL83]. The value of β together with its corresponding field distribution constitute a *mode* of the fibre. There are a countable number of modes with $n_{\text{CL}}k_0 < \beta < n_{\text{CO}}k_0$, and these are called guided modes.

Note that the propagation equation also has solutions with $0 < \beta < n_{\text{CL}}k_0$, *i.e.* outside the range where total internal reflection occurs. These modes can propagate in the cladding, and are called *radiative modes* [Mar91]. For these modes, the set of β values is infinite and continuous.

Each mode has a specific field distribution, with its own specific symmetry properties. From symmetry considerations it can be shown that for circularly symmetric fibres, modes can be either non degenerate or twofold degenerate [Isa75a; Isa75b]. In the latter case two field distributions with complementary symmetries are associated with the same propagation constant.

1.1.3 *Main properties*

1.1.3.1 *Number of modes*

The number of guided modes of the step index fibre depends on the dimensionless fibre V-parameter [SL83].

$$V = k_0\rho(n_{\text{CO}}^2 - n_{\text{CL}}^2)^{1/2}. \tag{1.2}$$

The smaller this parameter, the smaller the number of guided modes a fibre can carry. If at a given wavelength $V < 2.405$, a single degenerate pair of modes[2] is guided by the fibre. When there is only one possible value of β, the fibre is said to be *single-mode.* Either one of the pair of guided modes, associated with the same value of β, is referred to as the *fundamental mode.* Note that for a fibre to be single-mode, it needs to be designed with a combination of small core size to wavelength ratio and small difference in refractive indices between core and cladding. Conversely, a given fibre is always multi-mode for sufficiently small wavelengths. It is also worth noting that however small the parameter V is, there is always a fundamental guided mode of the fibre.

1.1.3.2 *Losses*

In the ideal step index fibre, the attenuation of a guided mode while propagating is solely due to material absorption so if the material is lossless then the guided mode is not attenuated at all. The intrinsic material absorption of pure silica for wavelengths between approximately 0.8 μm and 1.8 μm is very small (however in fabricated fibres, the absorption by the hydroxyl group OH is not negligible [Oko82] especially around its main peak centred at 1.38 μm), and in theory light in this wavelength range could be carried over hundreds of kilometers without noticeable loss. Nevertheless, until the early 1970s material absorption in fibres was considerable, due to contamination by water or metallic ions of the silica used to draw fibres. Since the work of Keck and his colleagues [KMS73], great improvements have been achieved in avoiding contamination and nowadays fibres can be drawn in which the attenuation of modes is no longer limited by absorption, but by Rayleigh scattering from nanoscopic fluctuations of the refractive indices [TSGS00]. This kind of fibre can have loss coefficients as low as 0.18 dB/km at $\lambda = 1.55$ μm, allowing the transmission of information over hundreds of kilometers without amplification. An historical survey of optical communication can be found in the first chapter of reference [Oko82].

Outside the low absorption wavelength range mentioned above, silica fibres are quite lossy. This is of no concern for telecommunications, where the minimum loss band has determined the wavelength used, but other applications in which guided optics beyond the minimum loss band would prove very useful, suffer from these limitations. Optical fibres operating with acceptable losses at the carbon dioxide laser wavelength (10.6 μm)

[2]This means that for a fixed wavelength one gets two different field maps associated with a single value of β. Waveguide symmetry and mode degeneracies will be studied in more details in Chapter 3.7.

would for example revolutionize industrial and surgical laser applications. Note that for high power light guidance, the limiting factor is not so much the power lost between the source and the target, but rather the temperature elevation in the fibre if losses are due to absorption, a high value of which can result in destruction of the waveguide.

1.1.3.3 *Dispersion*

In telecommunication networks, information is transmitted as binary data, taking the form of light pulses in optical fibres. In the field of optical waveguides, *dispersion* is a generic term referring to all phenomena causing these pulses to spread while propagating. There are essentially four causes of dispersion [Oko82; SL83]:

- *Inter-modal dispersion*: In a multi-mode fibre, for a given wavelength, different modes are associated with different values of β and hence different propagation velocities. This results, for a signal exciting more than one mode, in pulse spreading or echoing, depending on the propagation length. The obvious solution to avoid inter-modal dispersion is to use single-mode fibres.
- *Material dispersion*: All materials are intrinsically dispersive, *i.e.* the refractive index is wavelength dependent. Spectrally, a pulse of light is associated with a superposition of a range of frequencies, centered on the frequency of the modulated light source. Due to material dispersion, each spectral component of the pulse will propagate at different speeds, resulting in pulse spreading and deformation.
- *Waveguide dispersion*: Even without material dispersion, the solutions of the propagation equation are wavelength dependent: the propagation constant of a given mode is wavelength dependent. This leads to pulse spreading and deformation for the same reasons as above.
- *Polarization mode dispersion* [RU78; PW86; SP01]: It is in fact the same phenomenon as inter-modal dispersion, but the relevant modes are here originally degenerate. We have seen that a single-mode fibre in fact carries two degenerate modes. Because of anisotropic perturbations (stress, bends, torsion...) the degeneracy between these modes is *de facto* lifted, and inter-modal dispersion occurs. Until recently, the effects of polarization mode dispersion were negligible, but with the bit-rates and propagation lengths

aimed at today, and since other sources of dispersion can relatively easily be compensated for, polarization mode dispersion becomes a significant problem.

Chromatic dispersion (the so-called dispersion parameter D),[3] is the dispersion resulting from the combined effects of material and waveguide dispersion. When the chromatic dispersion D is positive (the dispersion regime is said to be anomalous), shorter wavelengths propagate faster than longer wavelengths. In the opposite case of negative D, the dispersion regime is said to be normal.

In optical telecommunications, to maximize bandwidth it is essential that light pulses keep their initial widths. Indeed, if light pulses spread, they eventually overlap and cannot be distinguished by the receiver. There are several ways of constraining the pulses to keep their initial widths. The first and most obvious is to design dispersionless fibres, through compensating material dispersion with waveguide dispersion. This is generally possible at only one wavelength, so that all information must be carried within a very narrow range of wavelengths. Shifting the zero-dispersion wavelength in silica fibres while keeping single-mode behaviour is generally achieved through sharp triangular index profiles of the core, with additional layers of different refractive indices between the core and the cladding ([AD86] and chapter 7 of reference [Oko82]). However, through such designs it is only possible to shift the zero-dispersion wavelength of 1.3 μm towards longer wavelengths.

A second way of keeping a constant pulse width during propagation is to use solitons, high power pulses for which the nonlinearity (more precisely the self phase modulation) of the fibre exactly compensates the dispersion. For solitons to exist, the fibre must have a relatively small anomalous dispersion.

A third method of keeping the pulse width constant is to retain a small, well known normal dispersion in the fibres, and to add dispersion compensating devices at each repeater. The latter can take the form of optical fibres with strong anomalous dispersion, which exactly compensate for the normal dispersion resulting from fibres between the repeaters. The reason for preferring the latter solution to optical fibres with strict zero dispersion

[3]Expanding the propagation constant β in a Taylor series around the pulsation ω_0, one obtains $\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + 0.5\,\beta_2(\omega - \omega_0)^2 + \ldots$ with $\beta_n = (\partial^n\beta/\partial\omega^n)_{\omega_0}$. β_0 is the inverse of the phase velocity, β_1 is the inverse of the group velocity, and β_2 is the group velocity dispersion parameter. The chromatic dispersion D is related to β_2 through the relation: $D = -(2\pi c\beta_2)/\lambda^2$.

is twofold. First, through careful design it is possible to obtain an effective zero-dispersion wavelength range (*i.e.* the range in which the combined effects of dispersion and dispersion compensation result in a negligible overall dispersion) that is much wider than the effective zero-dispersion wavelength range of zero-dispersion fibres. The wider available wavelength range enables wavelength multiplexing of information, *i.e.* encoding the signal using carrier waves having different wavelengths (*channels*) in a single fibre, which multiplies the bandwidth. Secondly, pulses carried in zero-dispersion optical fibres are subject to non-linear interactions, even if their power density is not large. Indeed since dispersion is negligible at the carrier wavelength, neighboring wavelengths will have a very long coupling length, thus increasing the effect of non-linear interactions. When using multiple wavelength channels, these undergo non-linear interactions (especially four-wave mixing) leading to channel crosstalk and information loss. State-of-the-art long haul telecommunication networks use fibres having small but non-zero, normal dispersion around $\lambda = 1.55$ μm,[4] and dispersion compensators at each repeater [Val01]. The operating wavelength $\lambda = 1.55$ μm was chosen for two reasons: it is the wavelength at which losses in silica are smallest, and it corresponds to the wavelength range in which erbium doped fibre light amplifiers are most effective. These light amplifiers [MRJP87; Des02] were invented relatively recently; they now replace electronic repeaters, enabling all-optical signal processing from source to receiver.

Regarding polarization mode dispersion, the usual way to avoid it is to use single-mode polarization maintaining fibres. Polarization maintaining fibres are optical fibres in which a birefringence has been introduced, generally through applying stress to the fibre during the drawing process or through using elliptical cores. The degeneracy between the two fundamental modes is then lifted, and since both modes have different polarization properties, they can be separated with polarizers by the receiver, eliminating polarization mode dispersion. In practice the whole process is not as straightforward as it sounds, and polarization maintaining fibres have numerous drawbacks. In order to keep a good coupling efficiency between two fibres or between fibres and devices, care must be taken to keep a precise alignment of the polarization axes, which is tricky given the approximate circular symmetry of the fibres. Further, even with the birefringence, with long propagation distances crosstalk between the two fundamental modes appears because of residual imperfections, so that polarization mode dis-

[4] These fibres are called Non-Zero Dispersion Shifted Fibres, or NZ-DSF.

persion is not completely eliminated. Single-mode single polarization fibres are fibres which carry only one polarization. They are produced in much the same way as polarization maintaining fibres, but they need a larger birefringence. Their main drawback is that the same physical effect that eliminates one of the two polarizations of the fundamental mode gives rise to leakage for the other polarization, so that low loss single polarization operation is not achievable with conventional optical fibres.

In the field of telecommunications, current work to further improve dispersion management – and hence bandwidth – is concentrated on obtaining fibres with flat near-zero dispersion, improvement of dispersion compensating devices and management of polarization mode dispersion.

For other fields of application other dispersion properties are sought. Anomalous and zero-dispersion at wavelengths below $\lambda = 1.3$ μm in single-mode fibres can be useful for super-continuum generation [WKOB$^+$00; RWS00; CCL$^+$01; DPG$^+$02; HH01], ultrashort pulse compression, soliton generation and propagation.

1.1.3.4 *Non-linearity*

Non-linear optical effects always appear when the power density of light is large enough, regardless of the material. Since in optical fibres light is well confined in a narrow core, non-linear effects can appear even in ordinary silica at relatively modest injection powers [Agr01]. These effects are generally a nuisance in long haul telecommunication networks, but can also be useful for some applications [Val01].

As far as the negative aspects are concerned [Chr90], there are three major non-linear effects in optical fibres arising from the Kerr effect (self phase modulation, cross-phase modulation, and four wave mixing), and two due to stimulated inelastic scattering (stimulated Brillouin and Raman scattering). We have already mentioned that four wave mixing leads to channel crosstalk in wavelength multiplexed systems. Self phase modulation and cross phase modulation result in chirping of the pulse frequency, which, combined with dispersion, gives rise to signal deformation and spreading, and hence crosstalk. Four wave mixing can be reduced through increasing the magnitude of dispersion in the fibre, whereas the penalty due to phase modulation effects tends to be less important if the dispersion is as small as possible.[5] Both stimulated scattering effects result in pulse power being

[5]Note that with large chromatic dispersion, interaction between channels is diminished and hence crosstalk modulation is reduced as well; there is no easy general rule

scattered into frequency-shifted waves. In the case of stimulated Brillouin scattering, the scattered wave propagates in the opposite direction to the incoming signal, and above a certain power threshold more power is scattered back than propagated forward. The stimulated Brillouin scattering threshold sets the upper limit of power which can be propagated efficiently through an optical fibre. In the case of stimulated Raman scattering, the scattered, frequency-shifted wave propagates in the same direction as the incoming signal. In wavelength multiplexed systems, the Raman scattered signal of one channel can hence overlap with another channel. Note that both scattering phenomena generally increase with fibre doping. All these effects are of course power dependent, and smaller power densities at the same injection powers can only be of benefit.

Most of the previous effects can also be exploited to create devices. When controlled, four wave mixing and cross phase modulation can be used for all-optical switching, frequency conversion, pulse reshaping and other forms of optical signal processing. Self phase modulation is essential for soliton propagation. Stimulated Raman scattering can be used for signal amplification [SI73]. The main restriction of applications using non-linearities of conventional step index optical fibres is that the material interacting with the light must be able to be drawn into a usable fibre. In practice this severely limits candidate materials, and most non-linear fibre applications either use the nonlinearities inherent in silica or nonlinear material that can be introduced into silica through doping.

1.2 Photonic Crystals

The idea of photonic crystals originated in 1987 from work in the field of strong localization of light [Joh87] and in the inhibition of spontaneous emission [Yab87]. It was subsequently shown that in periodic arrangements of –ideally lossless – dielectrics, the propagation of light can be totally suppressed at certain wavelengths, regardless of propagation direction and polarization. This inhibition does not result from absorption but rather from the periodicity of the arrangement, and is quite fundamental: in the frequency range where no propagation is possible (the so-called *photonic band gap*), the density of possible states for the light vanishes, so that

regarding the ideal dispersion properties to minimize cross phase modulation: Predicting and compensating cross phase modulation is far from easy and is a topic of current research.

even spontaneous emission becomes impossible. Such periodic arrangements of dielectrics have been called *photonic crystals*, or photonic band gap materials.[6] More background information can be found in the following book [JMW95].

1.2.1 *One dimension: Bragg mirrors*

The simplest device using the principles of photonic crystals is the one-dimensional photonic crystal, well known under the name of the Bragg mirror or the multilayer reflector. It consists of a periodic stack of two alternating dielectric layers. Light propagating in a direction normal to the layers undergoes successive reflection and transmission at each interface between adjacent layers. With an appropriate choice of layer thickness and refractive indices, waves reflected from each interface are in phase, whereas waves transmitted are out of phase. In that case, the transmitted wave components cancel each other out, and only the interference of the reflected components is contructive: the light is totally reflected. This works for a range of wavelengths. Bragg mirrors have been in use for decades, but it is only recently that they have come to be regarded as a special case of photonic crystals. The classical way of analysing Bragg mirrors with a finite number of layers, uses reflection and transmission matrices for each layer, and it is then quite straightforward to prove through recurrence relationships that reflection can be perfect with an infinite number of layers. There is nevertheless another approach to deal with a stack having an infinite number of layers, originating from solid state physics. If the stack is infinite, it has a discrete translational symmetry. The Bloch Theorem then applies, and solutions to the propagation equation in the stack are Bloch waves. Hence two wave vectors differing by a vector of the reciprocal lattice associated with the periodic stacking are physically the same: the dispersion diagram "folds back" along the limits of the Brillouin zone. At the edge of the Brillouin zone, two solutions exist having the same wave vector but different frequencies, and in between those two frequencies no solutions exist at all. The gap of frequencies for which no solutions exist is called a photonic band gap. Note that, until recently, reflection from Bragg mirrors was thought to be possible only within a relatively narrow range of angles of incidence. Recent work by Fink *et al.* has demonstrated the feasibility of omnidirectional reflection with Bragg mirrors [FWF+98].

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[6] Note that photonic crystals can also result from periodic arrangements of conductors.

1.2.2 *Photonic crystals in two and three dimensions*

Photonic crystals with two or three-dimensional periodicity can be seen as a generalization of Bragg mirrors. The simple approach with reflection and transmission matrices cannot be applied analytically here, and this is probably why their properties were discovered relatively recently, although, for example, important work on stacked grids for filtering in the far infrared was carried out by R. Ulrich in the 1960s [Ulr67; Ulr68]. The Bloch approach can be used similarly, and shows that band gaps can still open up. The point of using periodicities along two or three-dimensions is to open up an omnidirectional band gap: for the Bragg mirror, band gaps usually only exist for a narrow range of angles of incidence, and propagation parallel to the Bragg layers can never be inhibited. With photonic crystals having a two-dimensionally periodic arrangement of parallel rods, band gaps can exist for all directions of propagation in the plane of periodicity, and for photonic crystals with three-dimensional periodicity, propagation of light in all directions can be prohibited. When a band gap exists regardless of direction of propagation and polarization, one speaks of a total photonic band gap.

Photonic crystals with two-dimensional periodic arrangements are usually either made of parallel dielectric (or metallic) rods in air, or through drilling or etching holes in a dielectric material. In the field of integrated optics, holes of a fraction of a micrometer etched in slab waveguides are very promising for integrated photonic circuits, and have been succesfully demonstrated experimentally. Photonic crystals with three dimensional periodicity are a bit more tricky to achieve.[7] Yablonovitch suggested drilling an array of holes at three different angles into a dielectric material [YGL91]. The so-called wood-pile structure has attracted much attention [SD94; FL99; GdDT+03], and recent progress with artificial inverse opals is promising [BCG+00; VBSN01].

Note that the term photonic crystal was originally introduced to refer to materials having a photonic bandgap. It seems that it is now progressively more often used to refer to any kind of perodic arrangement of dielectrics or metals, with or without photonic band gaps. The latter generalization of the term makes sense considering that in solid state physics, a crystal is named so on account of the periodicity of its lattice, with band gaps ap-

[7] Nature, as so often, has demonstrated its supremacy in achieveing three dimensional photonic crystals billions of years before man. They can be found in opals. Note that one, two and three dimensional photonic crystals are also found elsewhere in nature: they give bright colours to beetles, butterflies, sea-mice and birds.

pearing in certain cases. Usual practice is then to reserve the term *photonic band gap material* for a photonic crystal having a photonic band gap. In the remaining chapters of the book we will avoid any confusion and speak of microstructures, since sometimes the dielectric structures may not even be periodic.

1.3 Guiding Light in a Fibre with Photonic Crystals

For frequencies within a total photonic bandgap, no propagation is allowed in an infinite photonic crystal. If a defect is introduced in the infinite lattice, localized defect states for isolated frequencies within the band gap can emerge, similar to bound states associated with defects in semiconductors. For three dimensional photonic crystal lattices, this can be a single point defect: in that case light emitted from within the defect will remain confined in the vicinity of the defect. It could also be a linear defect, in which case light remains in the locality of the defect but can propagate along it. Another way of looking at defect states is to consider the photonic crystal to be a perfect mirror in a certain frequency range. If one drills a hole all the way through the photonic crystal, light injected in the hole will be reflected at the borders of the hole and will propagate within it, in a similar way to that of light propagation in an optical fibre.

Given that photonic crystals can have a high reflection coefficient even with a relatively small number of periods, the width of the photonic crystal around the defect can be reduced to a few layers: we can hence imagine optical fibres consisting of a micrometric core surrounded by a photonic crystal cladding only a few times wider than the core. The resulting optical fibre is called a *photonic crystal fibre* and has an important difference from conventional optical fibres: in the latter, the core, in which light is guided, has to be of higher refractive index than the cladding. Using a photonic bandgap material for the cladding, reflection at some frequency is guaranteed regardless of the refractive index of the material inside the defect. A defect in a photonic crystal can hence confine and guide light in low refractive index media, such as a gas (air for example) or vacuum. This opens up possibilities never dreamt of before. An optical fibre guiding light in a vacuum would have absorption losses and non-linear effects reduced by orders of magnitude[8] compared with solid core fibres, paving the way for

[8]Material absorption, non-linear effects and material dispersion wouldn't *completely* vanish because of the evanescent parts of the fields remaining in the photonic crystal.

high power light guidance applications; material dispersion would become negligible, giving rise to completely new forms of dispersion management; guiding highly confined light in gas or liquids would enable the production of new types of non-linear fibres as well as a whole new family of fibre sensors; even guidance of atoms, molecules or cells through hollow core optical fibres would become possible [BKR02].

If one seeks to guide light along a linear defect, it is not necessary to use three dimensional periodicity or a total photonic band gap. Considering Fig. 1.1 but with the cladding now being a perfect mirror, $k_\perp$ will be set by the size of the defect, and β will follow from Eq. (1.1). If for the range of wave vectors given by these considerations no propagation is possible in the photonic crystal, a guided mode will exist. This can be achieved with two dimensional and even one dimensional radial (concentric) periodicities.

1.3.1 *Bragg fibres*

The idea behind the Bragg fibre is to use a Bragg mirror as a cladding of an optical fibre [YYM78]. Accordingly, a Bragg fibre is made out of a concentric arrangement of dielectrics, wound around a core which may or may not be hollow. Theoretical studies have shown only recently that hollow core Bragg fibres could be feasible, given the possibility of attaining large refractive index contrasts (*e.g.* 3 to 1.1 or 2.4 to 1.6, see Ref. [FWF$^+$98]) between successive layers. Indeed, the first working experimental hollow core Bragg fibre has already been demonstrated, and had in some wavelength ranges losses that are orders of magnitude smaller than any conventional optical fibre [THB$^+$02]. Furthermore, Bragg fibres can be single-mode, single-polarization even with full circular symmetry and without birefringence [Arg02; BA02].

1.3.2 *Photonic crystal fibres and hollow core microstructured optical fibres*

Strictly speaking, Bragg fibres are a special case of photonic crystal fibres. The term photonic crystal fibre is however mostly used to refer to fibres with a cladding consisting of a two dimensionally periodic array of inclusions. Their most immediate advantage over Bragg fibres is that lower refractive index contrasts are needed to achieve photonic bandgaps: a lattice of air (or vacuum) holes in silica or polymer is sufficient.

Fibres with a lattice of microscopic holes running along the fibre axis can be manufactured by drawing (*i.e.* heating and axially pulling) a preform containing macroscopic holes. The holey (*full of holes*) preform can readily be obtained either by stacking capillaries together, or through drilling (mainly for polymers) or by extrusion [vELA+01; Rus03]. In the drawing process, the overall shape of the preform is generally maintained, but the diameter of the cross-section is scaled down from centimetric to micrometric dimensions. Note that this process can be used to produce fibres with regular arrays of holes as well as fibres with non-periodic arrangements of holes, either with a solid or a hollow core (see Fig. 1.2). The general terms *holey fibre* and microstructured optical fibre (MOF) refer to any kind of fibre with a set of inclusions running along the fibre axis, whereas the term photonic crystal fibre is generally used to refer to MOFs in which guidance results from a photonic band gap effect. Note, however, that some authors also use the term *photonic crystal fibre* (PCF) for referring to MOFs in which the inclusions form a subset of a periodic array, but in which guidance may or may not result from photonic band gap effects[BBB03].

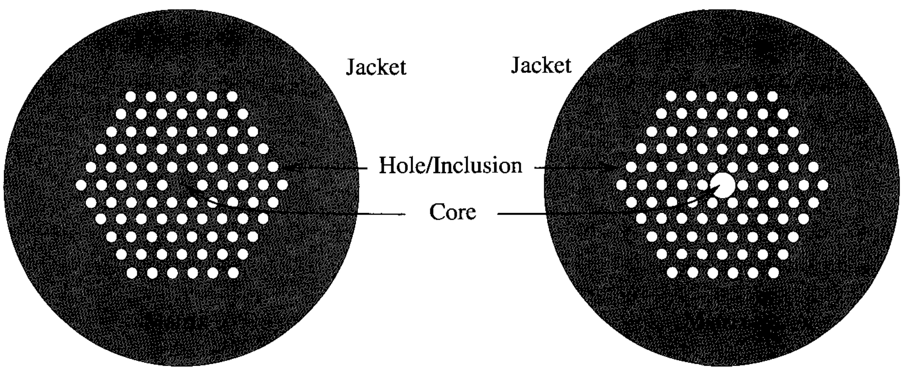

Fig. 1.2 Schematic representation of the cross-section of a typical solid core MOF (left) and hollow core MOF (right) with holes on a triangular lattice and a single central core. We will often refer to the microstructured part as the cladding. The jacket represents the physical boundaries of the MOF, and can be a solid jacket *e.g.* for mechanical protection or simply air. Note that the triangular lattice is also referred to as an hexagonal lattice.

1.3.2.1 *Hollow core MOFs*

Light guidance in hollow core MOFs can only be achieved by using the photonic band gap effect. Hollow core MOFs are hence necessarily photonic crystal fibres. We must consider a 2D photonic crystal with a 1D defect (due to the assumed translational invariance along the fibre axis z). The results concerning the band gaps are then quite different from those obtained in the usual case in which the longitudinal component of the wavevector, β, is equal to zero [MM94] as in thin-film nanophotonics in which the guidance occurs in a plane perpendicular to the axes of the inclusions. For hollow core MOFs, the most common configurations correspond to $|\beta|$ larger than both $|k_x|$ and $|k_y|$. It is then no longer necessary for the spatial period of the inclusion lattice to be of the order of half the wavelength: it can be much bigger, thus enlarging the possible parameter ranges yielding useful band gaps. For these photonic crystal fibres, the band gap is no longer a frequency range in which no propagation is possible irrespective of k_x and k_y but is instead a band gap in which for a given value of β no tranverse propagation is allowed.[9] For a silica matrix with a triangular lattice of circular air holes, there is no complete photonic band gap in the transverse propagation plane due to the low contrast in index between the two regions. Nevertheless, for $\beta \neq 0$ band gaps appear allowing the use of silica hollow core MOFs. Inserting a defect in the middle of the photonic crystal structure (which is infinite in most theoretical studies) will make possible the existence of a propagating mode in the perturbed crystal. If the propagation constant of this mode coincides with a band gap in the transverse plane then the mode will be confined in the locality of the defect, which constitutes the hollow core of the MOF.

These true photonic crystal fibres offer the whole range of benefits of hollow core fibres that we have described. Light guidance being possible solely within a photonic band gap, the wavelength range in which these fibres guide light is very narrow, only a few tens of nanometres for guidance in the infrared or the visible spectrum. Furthermore the accuracy of the periodicity of the lattice required to obtain a clear band gap effect makes the fabrication of these fibres challenging. Experimental hollow core MOFs have nonetheless been demonstrated using a cladding consisting of a triangular array of large air holes in silica [CMK+99], and the first the applications [BKR02] also make use of this basic structure.

[9] This explains why in hollow core MOF studies the band diagrams are given as a function of the (normalized) wavevector longitudinal component β.

1.4 Solid Core MOFs

1.4.1 *Guidance mechanism*

As we have already stated, photonic crystal fibre is only a particular case of MOF. The most common type of MOF is solid core MOF due to their (relative) ease of fabrication. For solid core MOFs it is often argued that guidance is due to *modified total internal reflection*: in a solid core MOF, the "average" refractive index of the cladding is lower than that of the core refractive index, leading to an equivalent geometry similar to those of conventional step index fibres. Following this argument, there is no need to invoke the concept of the photonic band gap, and any arrangement of holes – periodic or random – around a silica core results in a wave-guiding structure. It is easy to see the limits of such an interpretation of guidance, raising the question of what "average" means in this context: in the extreme case of a random hole distribution [MBBR00] with holes concentrating around a few isolated spots, leaving wide straight pathways for light to escape, it would be very surprising to find any kind of guidance. In fact, the "average" refractive index referred to when explaining guidance with modified total internal reflection is not a geometric average. It is actually not an average at all, but is a value extracted from the band structure of the surrounding arrangement of holes.[10] It corresponds to an "effective index" associated with the largest possible value of the propagation constant β for a given frequency in the microstructure: at a given frequency, light with a component β of the wave vector along the axis of the holes larger than a specific value β_{MAX} cannot propagate in the microstructured part of the fibre. This is analogous to total internal reflection in the case of step index fibres, in wich light with $\beta > n_{\text{CL}}k_0$ cannot propagate in the cladding. The average, or effective, index of the cladding of a MOF is then given by β_{MAX}/k_0. But since β_{MAX} is a band property, it is slightly contrived to distinguish the band gap between β_{MAX} and $\beta = \infty$ from any other band gap bounded by finite values of β. Modified total internal reflection can therefore also be seen as a specific case of band gap guidance. The only true difference with band gap guidance using other band gaps is that the band gap between β_{MAX} and $\beta = \infty$ always exists, regardless of frequency or the exact structure of the cladding, so that guidance relying on this band gap is much easier to achieve. Furthermore, this clarifies the idea that the effec-

[10]This kind of homogenization procedure implies to the artificial periodicising of the hole region.

tive index is in fact a property of the infinite periodic lattice surrounding the core, and thus the concept of modified total internal reflection must be used with circumspection when the holes around the core are not arranged periodically. The argument that modified total internal reflection guidance is not fundamentally different from band gap guidance is further discussed in Ref. [FSM+00a].

1.4.2 *Main properties and applications*

Guidance due to modified total internal reflection in solid core MOFs is much easier to achieve than band gap guidance, and indeed the first MOF in which guidance was demonstrated had a solid core [KA74; KBRA96]. All the new possibilities offered by photonic crystal fibres hitherto mentioned were based on the fact that guidance could be achieved in a hollow core, and guidance using photonic crystals in solid cores might seem uninteresting at a first sight. Nevertheless, the study of the first experimental solid core MOFs showed that these possess unique properties of their own, unachievable by conventional optical fibres. The most striking among these is certainly their ability to be single mode over an infinite range of wavelengths [KBRA96]. In other words, for some solid core MOFs, however small the wavelength is compared with the core size, only a single-mode is guided. This is fundamentally different from conventional fibres where, at small wavelength to core size ratios, multi-modedness is unavoidable. The importance of this property is not so much linked to the possibility of having single-mode guidance over a large range of wavelengths in the same fibre – most of the time the wavelength range at which a fibre will be used is quite narrow – but rather remains in the converse property: for a given range of wavelengths a solid core MOF with arbitrarily large core can be single-mode. Possibilities offered by the resulting large core single-mode fibres are unprecedented: in the field of telecommunications for example, where single-mode guidance is essential, if the core is larger then light can be injected with higher power without the power density reaching levels at which non-linear effects become problematic, so that the distance between repeaters can be greatly increased.

On the opposite side of the core size scale, it appears that because of the large index contrast modes are very well confined in the core, even when the wavelength to core size ratio is not small. This again differs from conventional fibres, in which the fraction of the field in the cladding at large wavelength to core size ratios is far from being negligible because of the very small difference in refractive index between the core and the cladding. Good confinement in small cores enables higher power densities and hence accentuated non-linear properties.

The large available parameter space of solid core MOFs (positions, sizes, and shapes of the holes, or refractive indices of the inclusions if they are not holes) makes the waveguide dispersion, which can have strong effects due to the high index contrast, highly configurable. Almost any dispersion curve seems accessible to MOFs with the correct design. The combination of endlessly single-mode guidance and adjustable dispersion has led to solid-core, single-mode MOFs with anomalous dispersion, to single-mode fibres with a zero-dispersion wavelength shifted down to the visible, as well as to single-mode fibres with ultra-flat normal or anomalous dispersion over a large wavelength range [FSMA00; KAB+00; RKR02; KRM03; RKM03]. With the additional possibility of good mode confinement, the configurable dispersion also gave rise to promising non-linear applications, either impossible to achieve with conventional fibres or having much lower power thresholds than in conventional fibres. These include temporal soliton formation and propagation [HH01; PBM+02; HGZ+02; WRW02], super-continuum generation [DPG+02; HH01; CCL+01; HGZ+02] and the formation of new types of stable spatial solitons [FZdC+03].

Finally, given the ease with which a defect may be introduced in the MOF lattice at the preform fabrication stage, MOFs with multiple cores or MOFs with large birefringence [OBKW+00] are straightforward to produce. Most MOFs consist of an array of holes in which one hole has been left out, playing the role of the core. With the symmetries of the lattices of holes that are generally used in practice (mostly six- or four-fold symmetries), this results in the fundamental mode being doubly degenerate, as in conventional optical fibres [SWdS+01]. If the core is now extended to two adjacent missing holes, or if the symmetry around the core is reduced to two-fold symmetry (*e.g.* through changing the sizes of two diametrically opposed holes [SKK+01], or with elliptical holes [SO01]), the degeneracy is lifted and the fibre becomes birefringent: the MOF becomes polarization maintaining. The resulting birefringence can be orders of magnitude larger than stress-induced birefringence in conventional polarization maintaining fibres, so that coupling between the two modes is greatly reduced. Single-mode single-polarization fibres can be achieved in the same way. We have already mentioned how important this could be in the elimination of polarization mode dispersion.

The main advantage of MOFs with multiple cores compared to conventional multiple core fibres is the ease with which they can be produced. If the cores are separated by a large number of lattice periods, the cores become independent, with negligible crosstalk, enabling, for example a spatial multiplexing of signals.[11] If on the contrary the cores are separated by only a few periods, modes guided in different cores are coupled. This can be useful *e.g.* for sensors, and bend sensors relying on multiple core MOFs have already been demonstrated [BBE+00; MGM+01]. Furthermore, through filling one or more holes with polymers, liquids, or gases, or through writing gratings into the MOF core, fibres with *in situ* adjustable properties as well as a great range of sensors and other devices can be obtained [EKW+01].

To sum up, although solid core MOFs do not seem as radically different from conventional fibres as hollow core MOFs at first sight, the range of new prospects they offer and the numerous fields in which they could exceed the performance of conventional fibres is at least as exciting as hollow core guidance. Furthermore, since guidance in solid core MOFs relies on modified total internal reflection[12] and not on the use of a very narrow band gap, solid core MOFs are also much easier to realize than hollow core MOFs. Solid core and hollow core MOFs are both extremely promising new types of fibres, with completely different properties and possible applications; given their huge differences there is not much point in comparing them directly.

1.5 Leaky Modes

1.5.1 *Confinement losses*

In the solid core MOFs that we will study, light guidance is due to modified total internal reflection between the core and a microstructured cladding consisting of inclusions in a matrix. The core and matrix material are generally the same, and hence have the same refractive index. In practice, the cladding has a finite width, as it consists of several rings of inclusions. Beyond the microstructured part of the fibre, the matrix extends without any inclusions until the jacket is reached. If we consider the jacket to be far from the cladding and core, and hence neglect its influence, guidance

[11] Note that the theoretical and practical feasibility of such multiplexing remains to be demonstrated.

[12] ...*i.e.* on a wide band gap existing for all materials.

in the core is solely due to a finite number of layers of holes in bulk silica extending to infinity. *A priori*, the cladding does not "insulate" the core from the surrounding matrix material since the holes do not merge with their neighbours and consequently the matrix is connected between the core and the exterior. Physically, we can imagine the light leaking from the core to the exterior matrix material through the bridges between holes, and thus expect losses. In the modified total internal reflection model of guidance, in which the microstructured part of the fibre is replaced by homogeneous material with an effective refractive index lower than the core, the core is completely surrounded by the cladding. The exterior matrix material and the core are then no longer directly connected. Nevertheless the width of the "effective cladding" is finite, and hence tunneling losses are unavoidable. Regardless of the approach one uses to explain guidance in MOFs, as long as guidance is due to a finite number of layers of holes, leakage from the core to the outer matrix material is unavoidable. We will call the losses due to the finite extent of the cladding *confinement losses*, or *geometric losses*.

1.5.2 *Modes of a leaky structure*

Confinement losses being unavoidable, modes of a MOF decay while propagating. They are no longer called guided modes, but *leaky modes* [Mar91; SL83]. The equations satisfied by the fields being linear, losses are proportional to the field intensity. Simple mathematics show that in that case the decay of the fields must be exponential along the direction of propagation. This is reflected by the propagation constant β taking an imaginary part. We have seen that modes are characterized by a transverse field distribution invariant with z, which is modulated by a phase factor taking the form $\exp(i\beta z)$. For leaky modes, β is complex, and the phase factor becomes $\exp(i\Re e(\beta)z)\exp(-\Im m(\beta)z)$ ($\Re e$ and $\Im m$ denoting the real and imaginary parts respectively). For the mode to decay in the direction of propagation (which is the case with leakage), the real and imaginary parts of β must be of the same sign.

This simple and elegant way of accounting for the decay of modes through a complex propagation constant is nevertheless only the tip of the iceberg of dealing with leaky modes. Indeed, using a complex propagation constant leads to more difficulties than simplifications. First, β being complex, the value of the perpendicular wavenumber $k_\perp$ (Eq. (1.1)) becomes complex as well. This has two consequences: it complicates the choice of the square root in Eq. (1.1), and the fields of the modes become divergent at large distances from the core. This in turn renders the modal field distributions non square-integrable, so that dealing with them becomes mathematically very delicate. As a consequence, leaky modes are not orthogonal in the usual sense and their completeness is not self-evident [SL83]. The very notion of modes becomes unclear, since the lack of orthogonality implies that two modes could interact: if for example the fundamental mode and the second mode have an intrinsic crosstalk, their distinction would become totally arbitrary. Finally we have seen that confined guided modes in the case of conventional step index fibres are found for values of β satisfying $n_{\text{CL}}k_0 < \beta < n_{\text{CO}}k_0$, and that for $0 < \beta < n_{\text{CL}}k_0$ there is a continuum of radiative modes. Since there is no natural ordering in the complex plane, we can no longer use such considerations to locate leaky modes. Furthermore, the continuum of radiative modes still exists and the range of β associated with radiative modes is not *a priori* disjoint from the range of β of leaky modes. This seriously complicates the task of finding leaky modes, since a leaky mode can be "lost" amongst a continuum of radiative solutions to the wave equations. Some of the difficulties pointed out here have not yet been overcome, and are the topic of current research (*cf.* paragraph 1.5.4). Since we refer to leaky modes in several parts of this book, we will nevertheless explain in more detail what leaky modes represent physically and mathematically, how some of the difficulties linked to their use can be circumvented, and more generally how we can justify our approach.

1.5.3 *Heuristic approach to physical properties of leaky modes*

To fully understand the slightly disconcerting properties of leaky modes, we have to keep in mind that modes are defined for a fibre that is *infinitely long.* The exponential decay of the mode with increasing z being equivalent to an exponential growth with decreasing z, the modal fields diverge when z approaches $-\infty$. We show here how this implies that the fields have to diverge radially along a cross-section of the fibre.

For the sake of simplicity, we consider a step index fibre consisting of a core with refractive index n_{CO} and a cladding of finite size with refractive n_{CL} surrounded by the same material as the core: beyond the cladding, the refractive index keeps a constant value n_{CO} everywhere (Fig. 1.3). In such a fibre, all modes are leaky because of tunneling losses [MY77; FV83]. We consider the fundamental (leaky) mode of that structure, prop-

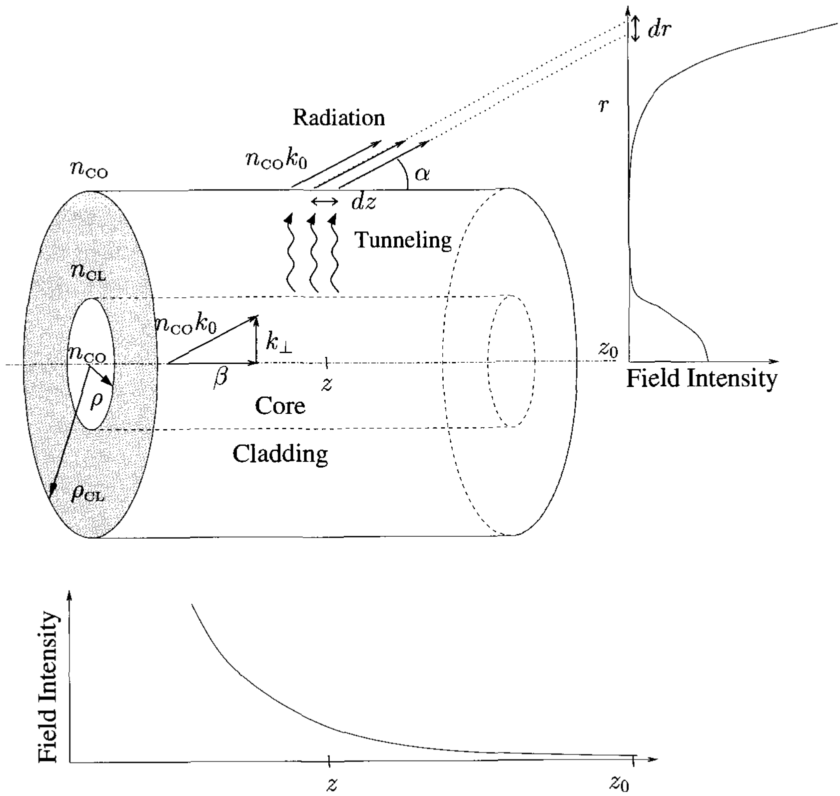

Fig. 1.3 Leaky modes and radial divergence, a heuristic approach. See text for details.

agating in the direction of increasing z. In the core, its power density distribution is similar to that of the fundamental mode of a lossless step index fibre. It is centrosymmetric with maximum value at the centre of the core. In the cladding the fields are evanescent, the power density decays exponentially with increasing distance from the core, until the exterior boundary of the cladding is reached. The propagation constant and the norm of the wave-vector being the same in the core and in the matrix surrounding the cladding, the amount of power which has reached the exterior cladding boundary can radiate away, and it does so at an angle of

$$\alpha = \cos^{-1}\left(\frac{\Re e(\beta)}{n_{CO}k_0}\right) . \tag{1.3}$$

In terms of rays, a radiated ray originating from the cladding at z will arrive at a reference position $z_0 > z$ at a radial distance

$$r(z) = \rho_{\text{CL}} + (z_0 - z)\tan\alpha \tag{1.4}$$

from the core centre. The whole power emitted from the cladding boundary in an infinitesimally long cylinder of length dz at z is found in the cross-section located at z_0 on an annulus with radius $r(z)$ and width $dr = dz\tan\alpha$. The power density in that annulus is hence the total power radiated from dz at z divided by the area of the annulus $2\pi r dr$. The total power radiated from the infinitesimal cylinder being proportional to the total power density at z and hence to $\exp(-2\Im m(\beta)z)dz$,[13] outside the cladding the power density $S(r)$ is proportional to

$$S(r) \propto \frac{1}{r\tan\alpha} \exp\left[-2\Im m(\beta)\left(z_0 - \frac{r-\rho_{\text{CL}}}{\tan\alpha}\right)\right]. \tag{1.5}$$

The power density at the center of the core $r = 0$ at z_0 being proportional to $\exp(-2\Im m(\beta)z_0)$, the normalized power density outside the cladding at z_0 is given by

$$\frac{S(r)}{S(0)} \propto \frac{1}{r\tan\alpha} \exp\left[2\Im m(\beta)\left(\frac{r-\rho_{\text{CL}}}{\tan\alpha}\right)\right]. \tag{1.6}$$

$\Im m(\beta)$ being positive, we see that the power diverges exponentially with increasing radial distance.

Counter-intuitive though it may seem, the radial exponential growth of field distributions is a fundamental property of leaky modes. Of course such a field distribution is impossible to obtain in practice. Real fibres are always of finite length, and hence by a reasoning similar to that given above will have to be limited by the value z_s of z at which the fibre starts. Upon incorporating this modification, we would find an exponential growth within the cross-section of the fibre, but the exponential growth would stop at $r(z_s)$, and for larger values of r the field would be strictly zero. The total power flowing through a cross-section at any given z would be equal to the power flowing through the cross-section at z_s. The exponential growth in the cross-section is not in contradiction with energy conservation: it is a direct consequence of it. This remains true in the case of infinitely long fibres, but the power flow through any cross-section is then infinite.

[13]The factor 2 in the exponential comes from the power being a square function of fields.

1.5.4 *Mathematical considerations*

In the example that we have considered above, we were led by intuition to the conclusion that the losses due to tunelling were small, and that their influence on the mode should thus remain small. Implicitly, we adapted the truly guided modes of a fibre with infinite cladding to kae amount of their lossy nature through letting β take a small imaginary part. Mathematically, this would amount to letting the lossless boundary conditions become slightly lossy without reformulating the whole problem. However, lossy boundary conditions, usually refered to as *open boundary conditions* are not mathematically straightforward to deal with.

Until recently, the only rigorous way of treating open boundary conditions was to avoid them: instead of considering the system as consisting solely of the core and the cladding, with lossy boundary conditions at the outer cladding boundary, we would consider the system consisting of the core, the cladding, and the rest of the universe, so that energy conservation is satisfied. The drawback of this approach is that the only modes which are solutions to the problem are associated with a continuum of real β values. The leaky modes, which allow us to analyse the physics of fibres with finite and infinite claddings along parallel lines, are not natural solutions to the physical problem encompassing the fibre and its exterior.

In contrast, the solutions to the true open boundary problem are the leaky modes. However, with open boundary conditions the system (the fibre alone) does not satisfy energy conservation. The mathematical operators used are then no longer hermitian, and the mathematics of non-hermitian operators is unpleasant. Firstly, eigenfunctions of non-hermitian operators do not form a complete orthogonal basis, but a set of non-orthogonal functions which may or may not be complete. Decomposing a field on this set is hence not straightforward, and the usual tools involving modal decomposition (*i.e.* almost all techniques in the theory of guided optics) cannot be used. Secondly, deriving rigorously the solutions to the physical problem raises difficulties. The work presented in this book is no exception in this regard, and the derivation we give of the multipole method is in fact not mathematically rigorous: for some key steps of the derivation, we implicitly assume the fields to be square integrable, yet leaky modes are not. More specifically, in the derivation of the Wijngaard identity, Appendix B.1, we use the Green's function

$$G_e = -\frac{i}{4} H_0^{(1)}(k_\perp^{\mathrm{M}} r). \tag{1.7}$$

The value of $k_\perp^{\text{M}}$ is not determined at that point of the derivation, but for leaky modes $k_\perp^{\text{M}}$ will be complex with a positive imaginary part, so that the Green's function will not be square integrable. The convolution used in the remainder of the derivation then becomes dubious. The derivation of the multipole method is rigorous for guided modes, but we cannot justify its extension to leaky modes with rigorous mathematics. However, the agreement between results obtained from the multipole method and results from other numerical methods or experiments, even when involving leaky modes, somewhat legitimates the confidence we have in the method.

Open boundary problems are a topic of current research. Recent work initiated by P. T. Leung and K. M. Pang [LP96; LTY97a; LTY97b; LSSY98; HLvdBY99; LLP99a; LLP99b; NLL02] suggests that the set of leaky modes of a class of open boundary problems, similar to the one considered here, forms a complete orthonormal basis if the space of functions and its inner product are adequately defined. From the point of view adopted in their work, the solution to the open boundary problem does not in fact include a continuum of eigenvalues, but solely the discrete, complete set of leaky modes. It is not yet clear if guidance in MOFs is strictly speaking a specific case of the class of problems studied by Leung *et al.*, but it might well be that the way to a rigorous derivation of multipole methods for leaky modes has already been paved.

1.5.5 *Spectral considerations*

There is a countable infinity of leaky modes [SL83; HLvdBY99]. For a step index fibre with infinite cladding, propagation constants satisfying $n_{\text{CL}} < \beta/k_0 < n_{\text{CO}}$ give strictly guided modes. The propagation constant of leaky modes being complex, we cannot use this argument any more. Nevertheless, we can assume that the "most confined" leaky modes of a fibre with finite cladding are similar to the guided modes of the fibre with the same parameters but with an infinite cladding, the main difference being that the propagation constant takes a small imaginary part. These modes would satisfy $n_{\text{CL}} < \Re e(\beta)/k_0 < n_{\text{CO}}$. For a solid core MOF, we could replace n_{CL} by the effective index of the cladding, as long as one can define such an effective index. Otherwise, another lower bound can be used, namely the refractive index of the inclusions n_{i}. Indeed, if $\Re e(\beta)/k_0 < n_{\text{i}}$, light is "likely to propagate" in the inclusions; there would be no barrier between the core and the exterior, and hence losses should be extremely high. For hollow core MOFs, light has to propagate in the hollow core and hence we must have $\Re e(\beta)/k_0 < 1$.

Note that when the imaginary part of β becomes very large, say one order of magnitude less than $\Re e(\beta)$, considerations relating to ordering become dangerous, and very leaky modes exist having values of $\Re e(\beta)$ well outside the mentioned boundaries. Finally, it is worth noting that having $n_{\rm CL} < \Re e(\beta)/k_0 < n_{\rm CO}$ does not imply that the imaginary part of β is small.

The fundamental mode of a step index fibre with infinite cladding is the mode with largest β. Consequently, it is also the mode with the fastest decaying evanescent tail in the cladding. For a microstructured fibre with a cladding of finite extent, we can thus expect the losses of the fundamental mode to be the smallest, so that its propagation constant would have both, the smallest $\Im m(\beta)$ and the largest $\Re e(\beta)$. When sufficient similarity between solid core MOFs and step index fibres with finite cladding exists, we can expect the same behaviour of the fundamental mode for MOFs. To locate the fundamental mode of a MOF it is therefore a good idea to start to look for values of β with small imaginary part, and with real part in the vicinity of $k_0 n_{\rm CO}$.