

Preface

The aim of this book is to introduce the concepts of critical phenomena and explore the common ground between complexity and criticality.

The word ‘complexity’ takes on a variety of meanings depending on the context, and its official definition is continuously being revised. This is because the study of complexity is in its infancy and is a rapidly developing field at the forefront of many areas of science including mathematics, physics, geophysics, economics and biology, to name just a few. Institutes and departments have been formed, conferences and workshops organised, books and countless articles written, all in the name of complexity. And yet, nobody agrees on a clear and concise theoretical formalism with which to study complexity. The danger is therefore that complexity research may become unstructured or even misleading. For our purposes, complexity refers to the repeated application of simple rules in systems with many degrees of freedom that gives rise to emergent behaviour not encoded in the rules themselves.

The word ‘criticality’, on the other hand, is well defined among statistical physicists. Criticality refers to the behaviour of extended systems at a phase transition where observables are scale free, that is, no characteristic scales exist for these observables. At a phase transition, the many constituent microscopic ‘parts’ give rise to macroscopic phenomena that cannot be understood by considering the laws obeyed by a single part alone. Criticality is therefore a cooperative feature emerging from the repeated application of the microscopic laws of a system of interacting ‘parts’. The phenomenology of phase transitions is well developed and there exists a sound theoretical formalism for its description.

The book is divided into three chapters. In the first two chapters, we carefully introduce the reader to the concepts of critical phenomena

using percolation and the Ising model as paradigmatic examples of isolated equilibrium systems. These systems undergo a phase transition only if an external agent finely tunes certain external parameters to particular values.

Percolation is one of the simplest models displaying a phase transition. The phase transition in percolation is purely geometrical and enables the reader to become intuitively familiar with important concepts such as fractals, scaling, and renormalisation.

The celebrated Ising model further develops the reader's intuition of emergent cooperative phenomena at a phase transition.

In the final chapter we consider the emergence of complexity in Nature. We argue that systems in Nature are neither isolated nor in equilibrium. We investigate a class of non-equilibrium systems where the constraint of having to tune external parameters to obtain critical behaviour is relaxed. We invite the reader to speculate on whether self-organisation in non-equilibrium systems may be a unifying concept for disparate fields such as statistical mechanics, geophysics and atmospheric physics.

Although mathematical methods have been developed to describe complexity and criticality, it is our experience that these methods are unfamiliar to scientists outside the field. Our hope is that this book will help students and researchers to treat complexity and criticality more quantitatively. Therefore, throughout the book we emphasise the mathematical quantitative techniques available.

The book is self-contained and therefore accessible to readers not familiar with the concepts of complexity and criticality. The text can form the basis for advanced undergraduate or graduate courses, and serve as an introductory reference for researchers in various fields. Each chapter is accompanied by exercises, full solutions to which can be obtained by contacting the authors via the book's associated website, <http://www.worldscibooks.com/physics/p365.html>. On this site, readers will also find animation codes to visualise the behaviour of the models considered. The bibliography is an attempt to cite the primary sources and most relevant publications but we readily acknowledge that it is by no means complete and might even be biased. We apologise if the reader is upset by any omissions or prejudice.

The book is based on a lecture course in statistical mechanics given at Imperial College London since 2000. We wish to pay tribute to all the students who attended the course. Their constructive feedback has been vital for the adapted presentation. In particular, we would like to thank Arno Proeme and Georg Schusteritsch for spotting many mistakes

and typos. We urge readers with an equally keen eye for details to contact the authors on encountering any errors.

We are grateful to Dimitri Vvedensky for suggesting the project in the first place. Furthermore, we thank Imperial College Press for being efficient and highly professional during the writing process and editor Katie Lydon for her seemingly infinite patience waiting for the final manuscript.

We are grateful to a vast number of people who have helped in one way or another throughout the writing process. In particular, present and former Ph.D. students at Imperial College London have been much involved: Nadia Farid, Vera Pancaldi, Ole Peters, Gunnar Pruessner, and Matthew Stapleton. Without their constant support and enthusiasm, we would never have finished the book. Indeed, the fantastic atmosphere in the condensed matter theory group at Imperial College London has been a fountain of inspiration. We also greatly appreciate the hospitality of PGP, University of Oslo, Norway, where parts of the book were written. Various insightful suggestions have been provided by Álvaro Corral and Jens G. Feder.

K.C. would like to pay a special tribute to Hugo S. Jensen, Hans C. Fogedby, Henrik J. Jensen, Per Bak, Zeev Olami, and Amnon Aharony. Hugo S. Jensen is a sublime high school physics teacher and he is really to blame for my becoming a physicist. Hans Fogedby's and Henrik Jensen's enthusiasm and insight into physics continues to amaze me. Besides being great physicists, they are great friends, as was Per Bak. Per Bak's passion and insight into science was second to none and Zeev Olami and I had some unforgettable years together with Per at Brookhaven National Laboratory. Thank you, Per and Zeev, for your inspiration, your limitless generosity and friendship. You are sorely missed and will always be remembered. K.C. was fortunate to attend a post-graduate course taught by Amnon Aharony on cooperative phenomena. The chapter on percolation is an attempt to live up to his high scientific standard and his pedagogical and formidable lectures. This book would never have materialised were it not for the great influence of these first rate physicists in my scientific life.

Finally, K.C. would like to mention his wonderful family, who have been tremendously supportive during this marathon; they never missed a chance to remind me that the manuscript was long overdue.

Kim Christensen and Nicholas R. Moloney