

Contents

<i>Preface</i>	vii
1. Basic Concepts	1
1.1 The Physics Simulation Process	3
1.2 Mesh Enhancement using Elliptic Methods.	9
1.3 A Brief Primer on Meshes	11
1.3.1 Mesh Topology	11
1.3.2 Structured Meshes	12
1.3.3 Unstructured Meshes	15
1.3.4 Basic Grid Generation Algorithms	16
1.3.4.1 Transfinite Interpolation Method (TFI)	17
1.3.4.2 Solution-Based Methods	18
1.3.4.3 Multiblock Method	19
1.3.4.4 Advancing Front Method	19
1.3.4.5 Delaunay Triangulation Method	20
1.3.4.6 Quadtree-Octree Methods	21
1.3.5 Mesh Quality Issues	23
2. Computational Geometry and Geometric Data Structures	25
2.1 Geometric Modeling	28
2.1.1 Curves	28
2.1.2 Properties of Parametric Curves	31
2.1.2.1 Arc-length	31
2.1.2.2 Curvature	31
2.1.2.3 Tangent Vectors	32
2.1.2.4 Principal Unit Normal	34

2.1.2.5	The Binormal Unit Vector	35
2.1.3	Constructing Parametric Curves	35
2.1.3.1	Wilson-Fowler Splines	39
2.1.3.2	Wilson-Fowler Splines for Closed Curves . .	49
2.1.3.3	The Parametric Cubic (PC) Curve	50
2.1.3.4	Operations on Curves	54
2.1.4	Parametric Surfaces	60
2.1.4.1	Ruled Surfaces	60
2.1.4.2	Translational Surfaces	61
2.1.4.3	Tensor Product Surfaces	63
2.1.4.4	The Parametric Bi-Cubic Patch: The 16- Point Surface	65
2.1.4.5	The Parametric Bi-Cubic Patch: The 4- Curve Surface	66
2.1.4.6	Parametric Surface Properties	69
2.1.4.7	Blending Patches at a Common Border . . .	72
2.1.4.8	Blending Between Non-Adjacent Patches . .	74
2.1.5	CSG Geometry	74
2.2	Mesh Operations On Geometry	77
2.2.1	Boundary Discretization	78
2.2.1.1	Equal Parameter-Increment Distribution . .	78
2.2.1.2	Equal Euclidean-Length Distribution	79
2.2.1.3	Equal Angle-Increment Distribution	81
2.2.2	Boundary Node Redistribution	84
2.3	Geometric Data Structures	89
2.3.1	IGES Data	89
2.3.2	Stereo Lithography	90
3.	Discretization Methods for Differential Equations	95
3.1	Finite Difference Methods	96
3.1.1	Finite Difference Operators	99
3.1.2	Non-uniform Difference Approximation	100
3.1.3	Finite Differences Applied to Differential Equations .	101
3.1.4	Consistency of Finite Difference Approximations . .	104
3.1.5	Stability of Finite Difference Approximations	105
3.1.6	Implicit Finite Difference Approximations	109
3.2	Weak Solutions	113
3.2.1	Hilbert Spaces	114
3.2.1.1	Functionals In Hilbert Spaces	118

3.2.1.2	Operators in Hilbert Spaces	120
3.2.2	Sobolev Spaces	124
3.2.3	Boundary Value Problem for Elliptic Operators . . .	126
3.2.3.1	The Dirichlet Boundary Value Problem . . .	130
3.2.3.2	The Neumann Boundary Value Problem . .	131
3.2.4	Weak Solution Methods	135
3.2.4.1	The Rayleigh-Ritz Method	135
3.2.4.2	The Galerkin Method	136
3.3	Applications of Weighted Residual Methods	138
3.3.1	The Subdomain Method	139
3.3.2	Finite Volume Methods	141
3.3.3	The Galerkin Method	150
3.4	The Finite Element Method	152
3.4.1	One-Dimensional Finite Element Methods	157
3.4.1.1	One-Dimensional Simplex Element Basis Functions	161
3.4.1.2	One-Dimensional Quadratic Element Inter- polation.	163
3.4.1.3	One-Dimensional Cubic Element Interpolation	171
3.4.2	Two-Dimensional Finite Element Methods.	178
3.4.2.1	A Note On Interpolation Error.	181
3.4.2.2	Two-Dimensional Bilinear Element Interpo- lation Using Quadrilateral Elements	181
3.4.2.3	Element Integrals: The Master Element . .	184
3.4.2.4	Two-Dimensional Linear Element Interpolat- ion: Triangular Elements	195
3.4.3	Three-Dimensional Elements	199
3.4.3.1	Tetrahedral Elements	199
3.4.3.2	Hexahedral or Rectangular Prism Elements	201
3.4.4	Computation of Element Matrices Using Gaussian Quadrature	202
3.4.5	A Brief Introduction to Errors in the Finite Element Method	204
4.	Solving the Mesh Enhancement Algebraic Equation System	209
4.1	Introduction	209
4.2	Selected Matrix Solution Methods	211
4.2.1	Jacobi Iterative Relaxation	213
4.2.2	Gauss-Seidel Relaxation	216

4.2.3	Other Iterative Approaches	216
4.3	Solution of Nonlinear Problems	221
4.3.1	Picard Linearization	222
4.3.2	Newton Linearization	223
4.4	Nonlinear Solution Methods for Finite Element Applications	226
4.4.1	Picard Linearization	227
4.4.2	Newton Linearization	228
4.4.2.1	Assembling the Tangent Stiffness Matrix (TSM)	230
4.4.2.2	Alternative Schemes of Addressing Nonlinear Terms	231
5.	The Geometry of Surfaces in Euclidean Space	235
5.1	The First Fundamental Form	236
5.2	The Second Fundamental Form	238
5.3	Intrinsic Equations of a Surface	244
5.3.1	The Gauss-Weingarten Equations	245
5.3.2	The Theorem of Gauss and the Codazzi Equations .	247
5.4	Covariant Differentiation	250
5.5	Geodesics in a Riemann Space	254
5.6	Curvature of Space	257
6.	Special Coordinate Systems	261
6.1	Isothermal Coordinates	262
6.1.1	Beltrami Equation	263
6.1.2	Complex Form	266
6.1.3	Solution of the Beltrami Equation	268
6.1.4	Teichmüller Spaces	270
6.2	Harmonic Coordinates	274
6.2.1	Sign Convention and Basic Formulas	277
6.2.2	The Ricci Tensor in Harmonic Coordinates	279
6.3	Conclusions	280
7.	Elliptic Mesh Enhancement Equation Systems	283
7.1	Khamayseh-Mastin Mesh Enhancement	284
7.2	Special Cases	287
7.2.1	Thompson-Thames-Mastin Generator	287
7.2.2	Winslow Grid Generator	290
7.3	Connection with Harmonic Coordinates	290

7.4	Variational Methods	291
7.5	Surface Meshes and Adaptive Methods	295
7.5.1	Numerical Examples	296
7.5.2	Adaptive Grid Methods	299
8.	Structured Mesh Smoothing and Enhancement	305
8.1	Introduction	305
8.1.1	Boundary Curve Discretization, Boundary Condi- tions, and Structured Terminology	309
8.1.2	Development of a Mesh Quality Metric	316
8.1.3	Introduction to Two-Dimensional Structured Mesh Methods	323
8.1.3.1	Transfinite Interpolation	326
8.1.3.2	Node Point Averaging Approaches	327
8.1.3.3	Winslow-Crowley Smoothing	335
8.2	Prescriptive Methods	345
8.2.1	Thompson-Thames-Mastin Generator	345
8.2.2	Specification of Control Functions <i>via</i> an Algebraic Transformation to a Parameter Space.	353
8.3	Laplace-Beltrami Mesh Enhancement Using a Target Metric	363
8.3.1	The Discrete Laplace-Beltrami System	368
8.3.2	Geometric Interpretation of the Metric Terms	372
8.3.3	The Target Metric Tensor	386
8.4	Using a Target Metric Surface to Implement Solution Adap- tivity	394
8.5	Rudimentary Methods for Unstructured Meshes	402
8.5.1	Laplace Smoothing	404
8.5.2	Ad hoc Laplace-Beltrami Methods	406
8.5.3	Winslow-based Unstructured Methods	408
8.6	Three-Dimensional Structured Laplace-Beltrami Methods	410
8.7	Conclusions	413
9.	Mesh Enhancement Methods for Unstructured Meshes	415
9.1	Laplace-Beltrami Enhancement Equations in Two and Three Dimensions	419
9.2	Approximation of the Laplace-Beltrami Equation System	422
9.2.1	Two-Dimensional Finite Element Implementation	424
9.2.2	Three-Dimensional Finite Element Implementation	428

9.2.3	Target Metric Tensor Estimation: Metric Equidistribution	430
9.2.4	Target Metric Tensor Estimation: Coarse Graining	433
9.2.5	Implementation Details and Summary	436
9.2.6	Two-Dimensional Examples	443
9.2.7	Three-Dimensional Examples	453
9.3	Quantitative Mesh Comparisons	465
9.4	Conclusions	469
Appendix A Differential Manifolds and Fiber Bundles		471
A.1	Scalar Field	472
A.2	Vectors and Tensors	473
A.3	Infinitesimal Transformations	475
A.4	Other Transformation Laws	477
A.5	Fiber Bundles	479
A.5.1	Differentiable Manifolds	480
A.5.2	Principal Fiber Bundles	486
A.5.3	Associated Fiber Bundles	487
A.6	Connections in Vector Bundles	489
A.7	Riemannian Manifolds	493
	<i>Bibliography</i>	501
	<i>Index</i>	511