# Preface

The use of special functions, and in particular of Airy functions, is rather common in physics. The reason may be found in the need, and even in the necessity, to express a physical phenomenon in terms of an effective and comprehensive analytical form for the whole scientific community. However, for almost the last twenty years, many physical problems have been resolved by computers. This trend is now becoming the norm as the importance of computers continues to grow. As a last resort, the special functions employed in physics will have, indeed, to be calculated numerically, even if the analytic formulation of physics is of first importance.

The knowledge on Airy functions was periodically the subject of many review articles. Generally these were about their tabulations for the numerical calculation of these functions which is particularly difficult. We shall quote the most known works in this field: the tables of J.C.P. Miller which are from 1946 and the chapter in the *Handbook of Mathematical Functions* by Abramowitz and Stegun whose first version appeared in 1954. No noteworthy compilation on Airy functions has been published since that time, in particular about the calculus implying these functions. For example, in the last editions of the tables of Gradshteyn and Ryzhik, they are hardly evoked. At the same time, many accumulated results in the scientific literature, remain extremely dispersed and fragmentary.

The Airy functions are used in many fields of physics, but the analytical outcomes that have been obtained are not (or weakly) transmitted between the various fields of research which after all remain isolated. Moreover the tables of Abramowitz and Stegun are still the only common reference to all the authors using these functions. Thus many of the results have been rediscovered, sometimes extremely old findings are the subject of publications and consequently a useless effort for researchers.

In this work, we would like to make a rather exhaustive compilation of the current knowledge on the analytical properties of Airy functions. In particular, the calculus implying the Airy functions is developed with care. This is, actually, one of the major objectives of this book. We are however aware of making a great number of repetitions regarding the previous compilations, but, it seemed necessary to ensure coherence. This book is addressed mainly to physicists (from undergraduate students to researchers). For the mathematical demonstrations, as one will see, we do not have any claim about the rigour.[1] The aim is the outcome, or the fastest way to reach it. Finally, in the second part of this work, the reader will find some applications to various fields of physics. These examples are not exhaustive. They are only given to succinctly illustrate the use of Airy functions in classical or in quantum physics. For instance, we point out to the physicist in fluid mechanics, that he can find what he is looking for, in the works of molecular physics or in physics of surfaces, and *vice versa.*

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[1]As a matter of fact, the Airy function can be considered as a distribution (generalised function) whose Fourier transform is an imaginary exponential. Also most of the integrals evoked in this work should be evaluated with the help of a convergence factor.