Chapter 1

A Historical Introduction : Sir George Biddell Airy

George Biddell Airy was born July 27, 1801 at Alnwick in Northumberland (North of England). His family was rather modest, but thanks to the generosity of his uncle Arthur Biddell, he went to study at Trinity College (University of Cambridge). He was a brilliant student although being a sizar,[1] and finally graduated in 1823 as a senior wrangler. Three years later, he was elected to occupy the celebrated Lucasian chair of mathematics. Nevertheless, his salary as Lucasian professor was too small to marry Richarda, as her father said. So he applied for a new position. In 1828, Airy obtained the Plumian chair becoming professor of Astronomy and director of the new observatory at Cambridge. His first works at this time were about the mass of Jupiter and also about the irregular motions of Earth and Venus.

In 1834, Airy started his first mathematical studies on the diffraction phenomenon and optics. Due to the diffraction phenomenon, the image of a point through a telescope is actually a spot surrounded by rings of smaller intensity, this spot is now called the "Airy spot", the associated Airy function has nothing to do with the purpose of the present book.

In June 1835, Airy became the 7^{th} Royal Astronomer and director of the Greenwich observatory, succeeding John Pond. Under his administration, modern equipment was installed, leading the observatory to its worldwide fame assisted by the quality of its published data. Airy also introduced the study of sun spots and the magnetism of Earth, he built a new apparatus for the observation of the Moon, and also for cataloging the stars. The question of absolute time was also a broad challenge, Airy defined the "Airy Transit Circle", that became in 1884: Greenwich Mean Time. But the

[1] With the meaning that he paid a reduced fee but worked as a servant to richer students.

renown of Airy is also due to the "Neptune affair". During the decade 1830–40, astronomers were interested in the perturbations of Uranus that were discovered in 1781. In France, François Arago suggested to Urbain Le Verrier finding a new planet that might cause the perturbations of Uranus. In England, the young John Adams was doing the same calculations with a slight advance, however Airy was doubtful on the issue of such a work. Adams tried twice to meet Airy in 1845 but was unsuccessful: the first time Airy was away, the second time Airy was taking dinner and did not like to be disturbed. Finally, Airy entrusted the astronomer James Challis with the observation of the new planet from the calculations of Adams. Unfortunately, Challis failed in his task. At the same time, Le Verrier asked the German astronomer Johann Galle in Berlin to locate the planet from his data: the new planet was discovered on September 20, 1846. A polemic started then between Airy and Arago, between France and England, and also against Airy himself. The polemic spread out with the name of the planet itself: Airy wanting to name the new planet Oceanus. The name of Neptune was finally given. The story goes that in the end, Adams and Le Verrier became good friends.

In 1854 Airy attempted to determine the mean density of the Earth. The experiment stood in the comparison of gravity forces on a single pendulum at the entrance of a pit and at its ground. This experiment was carried out near South Shields in a mine of 1250 feet in depth. Taking into account the elliptical form and the rotation of the Earth, Airy found a density of 6.56, which is not so far – considering the epoch – from the usually admitted density 5.42.

Airy was knighted in 1872, and so became Sir George Biddell.[2] At this time, Airy started a lunar theory. The results were published in 1886, but in 1890 he found an error in his calculations. The author was eighty-nine years old and was unwilling to revise his calculations. Late in 1881, Sir George left his astronomer position at Greenwich for retirement. He died January 2, 1892.

The autobiography of Sir George, edited by his son Wilfred, was published in 1896 ("*Autobiography of G.B.* AIRY", W. Airy ed., 1896). The name of Airy is associated with many phenomena such as the Airy spiral (optical phenomenon visible in quartz crystals), the Airy spot in diffraction phenomena or the Airy stress function he introduced in his work on elasticity, different as well from the Airy functions that we shall discuss in this

[2] After he declined the offer on three occasions, arguing the fees.

Fig. 1.1 Sir George Biddell Airy (after the Daily Graphic, January 6, 1892).

book. Among of the most-known books he wrote, we may quote "*Mathematical tracts on physical astronomy*" (1826) and "*Popular astronomy*" (1849).

Airy was particularly involved in optics, for instance he made special glasses to correct his own astigmatism. For the same reason, he was also interested by the calculation of light intensity in the neighbourhood of a caustic [Airy (1838), (1849)]. For this purpose, he introduced the function defined by the integral

$$W(m) = \int_0^\infty \cos\left[\frac{\pi}{2}\left(\omega^3 - m\omega\right)\right] \mathrm{d}\omega,$$

which is now called the Airy function. This is the object of the present book. W is the solution of the differential equation

$$W'' = -\frac{\pi^2}{12} mW.$$

The numerical calculation of Airy functions is somewhat tricky, even today!

However in 1838, Airy gave a table of the values of W for m varying from -4.0 to $+4.0$. Thence in 1849, he published a second table for m varying from -5.6 to $+5.6$, for which he employed the ascending series. The problem is that this series is slowly convergent as m increases. A few years after, Stokes (1851, 1858) introduced the asymptotic series of $W(m)$, of its derivative and of the zeros. Practically no reseach was endeavoured on Airy function until the work of Nicholson (1909), Brillouin (1916) and Kramers (1926) who contributed broadly to a better knowledge of this function.

In 1928 Jeffreys introduced the notation used nowadays

$$Ai(x) = \frac{1}{\pi} \int_0^{\infty} \cos\left(\frac{t^3}{3} + xt\right) dt,$$

which is the solution of the homogeneous differential equation, called the Airy's equation

$$y'' = xy.$$

Clearly, this equation may be considered as an approximation of the differential equation of the second order

$$y'' + F(x)y = 0,$$

where F is any function of x. If $F(x)$ is expanded in a neighbourhood of a point $x = x_0$, we have to the first order $(F'(x_0) \neq 0)$

$$y'' + [F(x_0) + (x - x_0)F'(x_0)]\, y = 0.$$

Then with a change of variable, we find the Airy's equation. This method is particularly useful in a neighbourhood of a zero of $F(x)$. The point x_0 defined by the relation $F(x_0) = 0$ is called a transition point by mathematicians and a turning point by physicists. Turning points are involved in the asymptotic solutions of linear differential equations of the second order [Jeffreys (1942)], such as the stationary Schrödinger equation.

Finally we can note that Airy functions are Bessel functions (or linear combinations of these functions) of order 1/3. The relation between both of the Airy's equation and the Bessel equation is done with the change of variable $\xi = \frac{2}{3}x^{3/2}$, yielding Jeffreys (1942) to say: "*Bessel functions of order* 1/3 *seem to have no application except to provide an inconvenient way of expressing this function*"!