

# Chapter 1

## Introduction

### 1.1 Exordium

Periodic focusing accelerators, transport systems and storage rings [1–16] have a wide range of applications ranging from basic scientific research in high energy and nuclear physics, to applications such as spallation neutron sources, heavy ion fusion, tritium production, and nuclear waste treatment. Of particular importance, at the high beam currents and charge densities of practical interest, are the effects of the intense self fields produced by the beam space charge and current on determining the detailed equilibrium, stability and transport properties, and the nonlinear dynamics of the system. Intense charged particle beams (or charge bunches), like one-component non-neutral plasmas [1], are a many-body collection of charged particles which exhibit a broad range of collective phenomena, such as plasma waves and instabilities. Moreover, the intense self fields in a charged particle beam can have a large influence on the detailed dynamics and stability behavior of the beam.

At the beam intensities of practical interest for present and next-generation accelerators and transport systems, it is increasingly important to develop an improved theoretical understanding of collective processes affecting intense beam propagation through periodic focusing field configurations. Making use of theoretical models and techniques developed in the description of one-component non-neutral plasmas [1, 17–22] and multispecies electrically-neutral plas-

mas [23–30], this book provides a systematic treatment of the nonlinear dynamics and collective processes in intense charged particle beams. For example, one such statistical framework for describing intense beam propagation is based on the nonlinear Vlasov-Maxwell equations. Such a kinetic model describes the self-consistent nonlinear evolution of the beam distribution function  $f_b(\mathbf{x}, \mathbf{p}, t)$  in the six-dimensional phase space  $(\mathbf{x}, \mathbf{p})$  as the beam particles interact with the applied focusing field configuration and the average self-generated electric and magnetic fields,  $\mathbf{E}^s(\mathbf{x}, t)$  and  $\mathbf{B}^s(\mathbf{x}, t)$ , produced by the beam space charge and current. Through analytical studies based on the nonlinear Vlasov-Maxwell equations, and numerical simulations using particle-in-cell models and nonlinear perturbative simulation techniques, considerable progress has been made in developing an improved understanding of the collective processes and nonlinear beam dynamics characteristic of high-intensity beam propagation in periodic focusing and uniform focusing transport systems [31–57]. The Vlasov-Maxwell formalism is described in Chapter 2 of this book, and forms the basis for the analysis in Chapters 4–8. Such an approach constitutes a complete theoretical description of the nonlinear dynamics and collective processes in intense charged particle beams, and is particularly useful in describing phenomena that depend on detailed properties of the distribution function  $f_b(\mathbf{x}, \mathbf{p}, t)$  in the six-dimensional phase space  $(\mathbf{x}, \mathbf{p})$ .

An alternative theoretical formalism, based on the macroscopic fluid-Maxwell equations [58–69], is developed in Chapter 2 and applied to investigate collective oscillations and instabilities driven by pressure anisotropy in Chapter 9. Such a macroscopic model treats the charged particle beam as a fluid, and follows the nonlinear evolution of the macroscopic beam properties such as the number density of beam particles,  $n_b(\mathbf{x}, t) = \int d^3p f_b(\mathbf{x}, \mathbf{p}, t)$ , the average flow velocity,  $\mathbf{V}_b(\mathbf{x}, t) = (\int d^3p f_b)^{-1} \int d^3p \mathbf{v} f_b(\mathbf{x}, \mathbf{p}, t)$ , where  $\mathbf{v} = \mathbf{p}/\gamma m_b$  is the velocity and  $\gamma = (1 + \mathbf{p}^2/m_b^2 c^2)^{1/2}$  is the relativistic mass factor, and the particle pressure tensor  $\mathcal{P}(\mathbf{x}, t)$ . Closure of the coupled fluid equations is typically achieved by neglecting the effects of heat flow. Such a macroscopic model is inherently simpler than a fully kinetic treatment, but also has less physics content, such as detailed information regarding the distribution of beam particles in phase space.

Nonetheless, a macroscopic fluid model is found to provide many useful insights into collective processes occurring in intense charged particle beams.

In summary, the present volume has been prepared as a graduate-level text which covers a broad range of topics related to the fundamental properties of the collective processes and nonlinear dynamics of intense charged particle beams. The subject matter is treated systematically from first principles using a unified theoretical approach, and the emphasis is on the development of basic concepts that illustrate the underlying physical processes in circumstances where intense self fields play a major role in determining the evolution of the system. The statistical models used to describe the properties of intense charged particle beams are based on the Vlasov-Maxwell equations, the macroscopic fluid-Maxwell equations, or the Klimontovich-Maxwell equations, as appropriate, and extensive use is made of theoretical techniques developed in the description of one-component nonneutral plasmas [1, 17–22], and multispecies electrically-neutral plasmas [23–30], as well as established techniques in classical mechanics, electrodynamics and statistical physics [70–75]. These approaches are found to provide remarkably tractable and robust theoretical frameworks for describing the detailed nonlinear dynamics and collective processes in intense charged particle beams, including regimes where the intense self-fields produced by the beam space charge and current play a major role in determining the detailed evolution of the system.

## 1.2 Historical Background

As noted in Sec. 1.1, this book builds on the theoretical models and techniques developed in the description of one-component nonneutral plasmas, and extends and applies these methods to describe the nonlinear dynamics and collective processes in intense charged particle beams. The early research in these two subfields of physics proceeded largely independently, and have interesting similarities and differences. In both cases, the very early research on one-component nonneutral plasmas and charged particle beams predated, by several decades, common usage of the terms ‘plasma’ or ‘nonneutral plasma’

in the lexicon of modern-day physics. (The term ‘plasma’ was introduced by Tonks and Langmuir [76] in 1929 to describe collective electron plasma oscillations in an ionized gas, although widespread use of this descriptor did not occur until the 1960s.) Indeed, the classic papers by Child [77], Langmuir [78], Brillouin [79], Pierce [80], Kyhl and Webster [81], and Buneman [82], represent some of the earliest efforts to investigate theoretically and experimentally the equilibrium and stability properties of nonneutral electron flow in planar diodes and in geometries with crossed electric and magnetic fields. On the other hand, stimulated by the classic work of Courant, Livingstone and Snyder [83, 84], and Christofilis [85], applied periodic (alternating-gradient) focusing fields have been used for the propagation and acceleration of charged particle beams [1–14] since the early 1950’s, ranging from storage rings and cyclic accelerators such as the synchrotron [11, 14, 86], used in high energy and nuclear physics, to periodic focusing quadrupole lattices used in linear induction accelerators for heavy ion fusion applications [15, 16]. Budker’s classic work [87] on relativistic stabilized electron beams also occurred during this early period.

In both cases, the early research on one-component nonneutral plasmas and charged particle beams predated the major international development of the theoretical foundations of modern plasma physics, which began to a large extent during the 1960’s. Moreover, advances in the basic understanding of one-component nonneutral plasmas and the dynamics of charged particle beams during this early period appear to have proceeded largely uninfluenced by the seminal works of Vlasov [88], Landau [89] and Bogoliubov [90] during the 1940’s on collective interactions in many-body charged particle systems. For the case of one-component nonneutral plasmas, this is likely due to the fact that the emphasis during this early period was mainly on the practical use and control of space-charge waves on nonneutral electron beams in microwave generation devices (such as klystrons, traveling wave tubes and magnetrons) and vacuum tube diodes. For the case of charged particle beams used in high energy and nuclear physics applications, this is likely due to the fact that during this early period the achievable beam currents and charge densities were sufficiently low that collective processes were relatively weak, and single-particle

models (or simple superpositions thereof) provided an adequate description of the beam dynamics. While there are fascinating aspects of the early development of these two subfields of physics, this is not the main subject matter of this book, and the reader is referred to the extensive bibliographies in Refs. 1–13 for further details.

During the past decade, the common physics of one-component nonneutral plasmas and intense charged particle beams has become increasingly evident. Indeed, many important scientific advances have been made, both in terms of theoretical understanding and instrumentation techniques, since the properties of intense charged particle beams and laboratory-confined nonneutral plasmas were investigated using similar theoretical formalisms in *Physics of Nonneutral Plasmas* (Addison Wesley, 1990) [1]. Of course an intense charged particle beam (or charge bunch), when considered in the beam frame, is in fact a one-component nonneutral plasma [1], which would be expected to exhibit many collective properties similar to laboratory-confined pure ion plasmas or pure electron plasmas. In this regard, there is an extensive literature on the collective properties and nonlinear dynamics of laboratory-confined nonneutral plasmas, both in Paul-trap configurations [91–96], and in Malmberg-Penning trap configurations [1, 17–21], which are relevant to the general subject matter of this book. While not covered here, areas of basic research on one-component nonneutral plasmas [1] include (for example): the development of precision atomic clocks [97, 98]; research on properties of strongly correlated (including crystalline) one-component nonneutral plasmas [17, 19, 99–102]; investigations of nonlinear vortex dynamics and turbulence in nearly-inviscid two-dimensional fluid flow [19, 103, 104]; studies of particle transport across magnetic field lines [19, 105, 106]; investigations of the effects of electron-neutral collisions on cross-field particle transport in a pure electron plasma [19, 107–109]; studies of the thermal equilibrium and thermodynamic properties of one-component nonneutral plasmas [17, 18]; research on the formation and confinement properties of positron plasmas [19, 110, 111]; and investigations of the production of antihydrogen for basic physics studies by the mixing of positron and antiproton plasmas [19, 112–114]. While a detailed discussion of these topics is beyond the scope of a treatise on intense charged particle beams, it

should be emphasized that many of the theoretical techniques and collective properties of laboratory-confined one-component nonneutral plasmas are relevant to the general subject matter treated in subsequent chapters.

Finally, as noted earlier, the principal focus of this book is on the nonlinear dynamics and collective processes in intense charged particle beams propagating in periodic focusing field configurations. In this regard, two important topics and applications areas of charged particle beams are not included in the present treatise. One topic is the generation of coherent radiation by energetic electrons in free electron devices [1, 115–123] such as free electron lasers, gyrotrons and cyclotron masers, klystrons and gyroklystrons, and magnetron configurations. A second important topic not covered in subsequent chapters is that of advanced accelerator concepts [124–129], ranging from the plasma wakefield accelerator, to the beat-wave accelerator, to various novel laser-plasma acceleration techniques, which offer the potential of very high acceleration gradients over short distances (GV/m) relative to conventional approaches (several MV/m). Many of the theoretical techniques developed in subsequent chapters can be applied to both of these topics.

### 1.3 Characteristic Frequencies

Detailed theoretical frameworks for describing the nonlinear dynamics and collective processes in intense charged particle beams are developed and applied in Chapters 2–10. In this section, to orient the reader, we provide a brief summary of the characteristic frequencies that describe the motion of a test particle in an intense charged particle beam. For present purposes, we consider an axially continuous beam with characteristic root-mean-square radius  $R_b$  propagating in the  $z$ -direction with average axial velocity  $V_b = \beta_b c = \text{const.}$  through a periodic focusing lattice with axial periodicity length  $S \gg R_b$ . The directed axial kinetic energy is  $(\gamma_b - 1)m_b c^2$ , where  $m_b$  is the rest mass of a beam particle,  $c$  is the speed of light in *vacuo*, and  $\gamma_b = (1 - \beta_b^2)^{-1/2}$  is the relativistic mass factor. The charge of a beam particle is denoted by  $e_b$ , the characteristic number density of beam particles is denoted by  $\hat{n}_b$ , the characteristic thermal speed associated

with the (random) transverse motion of a beam particle is denoted by  $v_{Tb} = \langle v_x^2 + v_y^2 \rangle^{1/2}$ , and the particle motion in the beam frame is assumed to be nonrelativistic. Here,  $\langle \chi \rangle \equiv N_b^{-1} (\int dx dy d^3 p \chi f_b)$  denotes the statistical average of a phase function  $\chi$  over the distribution function  $f_b$ , and  $N_b \equiv \int dx dy d^3 p f_b$  is the number of beam particles per unit axial length. For our purposes here, we adopt a *smooth-focusing* model in which the applied transverse focusing force on a beam particle is specified by

$$\mathbf{F}_{foc} = -\gamma_b m_b \omega_{\beta\perp}^2 (x \hat{\mathbf{e}}_x + y \hat{\mathbf{e}}_y) . \quad (1.1)$$

Here,  $\mathbf{x}_\perp = x \hat{\mathbf{e}}_x + y \hat{\mathbf{e}}_y$  is the transverse displacement of a particle from the beam axis at  $(x, y) = (0, 0)$ , and  $\omega_{\beta\perp} = \text{const.}$  is the characteristic transverse oscillation frequency of a beam particle in the applied focusing field.

The characteristic frequencies (inverse time scales) that describe the motion of a test particle in the beam are given by:

(a) The frequency  $\omega_s$  associated with the axial transit of a beam particle over one lattice period,

$$\omega_s = \frac{2\pi V_b}{S} , \quad (1.2)$$

(b) The frequency  $\omega_{\beta\perp}$  associated with the transverse motion of a single particle in the applied focusing field,

$$\omega_{\beta\perp} , \quad (1.3)$$

(c) The collective oscillation frequency  $(\hat{\omega}_{pb}^2 / 2\gamma_b^2)^{1/2}$  associated with the self-field effects produced by the beam space charge and current,

$$\frac{1}{(2\gamma_b^2)^{1/2}} \hat{\omega}_{pb} = \frac{1}{(2\gamma_b^2)^{1/2}} \left( \frac{4\pi \hat{n}_b e_b^2}{\gamma_b m_b} \right)^{1/2} , \quad (1.4)$$

(d) The frequency  $\omega_\epsilon$  associated with the transit of a thermal particle over the transverse dimension of the beam

$$\omega_\epsilon = \frac{v_{Tb}}{R_b} . \quad (1.5)$$

In Eq. (1.4), the quantity  $\hat{\omega}_{pb} = (4\pi\hat{n}_b e_b^2 / \gamma_b m_b)^{1/2}$  is referred to as the plasma frequency [1, 23–25]. Note that  $\hat{\omega}_{pb}^2$  is directly proportional to the number density of beam particles ( $\hat{n}_b$ ), which in turn is proportional to the (repulsive) transverse electric force on a beam particle due to the beam space charge. The  $-\beta_b^2$  contribution to the factor  $1/\gamma_b^2 = 1 - \beta_b^2$  occurring in Eq. (1.4) is associated with the fact that the (focusing) self-magnetic force produced by the axial beam current *reduces* the combined self-electric plus self-magnetic force on a beam particle. The factor  $1/\sqrt{2}$  occurring in Eq. (1.4) is a geometric factor which results when a rigorous calculation of the collective oscillation frequency is carried out in Chapters 3 and 5.

In practical applications, the characteristic frequencies  $\omega_s$  and  $\omega_{\beta\perp}$  defined in Eqs. (1.2) and (1.3) are comparable in size. Indeed, a vacuum phase advance  $\sigma_v < \pi/2$  corresponds to the inequality  $\omega_{\beta\perp} < (1/4)\omega_s$  (Chapter 3). In addition,  $\omega_{\beta\perp}^2$  is proportional to the applied focusing force, whereas  $\hat{\omega}_{pb}^2/2\gamma_b^2$  is proportional to the *net* self-field force on a beam particle, which is *defocusing*. Therefore, for transverse confinement of a beam particle by the applied focusing field, we require that  $\hat{\omega}_{pb}^2/2\gamma_b^2 < \omega_{\beta\perp}^2$ , or equivalently,

$$s_b \equiv \frac{\hat{\omega}_{pb}^2}{2\gamma_b^2 \omega_{\beta\perp}^2} < 1. \quad (1.6)$$

In Eq. (1.6), the dimensionless quantity  $s_b$  is a convenient measure of the normalized beam intensity and the dc self-field force on a beam particle. Two limiting regimes of charged particle beam propagation are readily characterized in terms of the parameter  $s_b$  and the frequencies defined in Eqs. (1.4) and (1.5). The first corresponds to a *very-low-intensity, emittance-dominated* beam with

$$s_b \ll 1, \quad (1.7)$$

and  $\hat{\omega}_{pb}^2/2\gamma_b^2 \ll \omega_\epsilon^2$ , or equivalently,

$$\frac{\hat{\omega}_{pb}^2 R_b^2}{2\gamma_b^2} \ll v_{Tb}^2. \quad (1.8)$$

The second corresponds to a *very-low-emittance, space-charge-dominated* beam with

$$s_b \rightarrow 1, \quad (1.9)$$

and  $\hat{\omega}_{pb}^2/2\gamma_b^2 \gg \omega_\epsilon^2$ , or equivalently,

$$\frac{\hat{\omega}_{pb}^2 R_b^2}{2\gamma_b^2} \gg v_{Tb}^2. \quad (1.10)$$

The conditions in Eqs. (1.9) and (1.10) are characteristic of the high-intensity beams used for heavy ion fusion applications. On the other hand, the system parameters characteristic of present and next-generation accelerator facilities for high energy and nuclear physics applications are typically intermediate between the conditions in Eqs. (1.7) and (1.8) and the conditions in Eqs. (1.9) and (1.10). For example, the nominal value of normalized beam intensity is  $s_b \simeq 0.2$  for the proton beams envisioned in the Spallation Neutron Source storage ring, and the normalized beam intensity is  $s_b \simeq 0.15$  in Fermilab's Tevatron accelerator at the injection phase. Finally, if we define an effective transverse Debye length by  $\lambda_{D\perp} \equiv \sqrt{2\gamma_b v_{Tb}/\hat{\omega}_{pb}}$ , note that Eq. (1.10) corresponds to  $\lambda_{D\perp} \ll R_b$  (for  $s_b \rightarrow 1$ ), whereas Eq. (1.8) corresponds to  $\lambda_{D\perp} \gg R_b$  (for  $s_b \ll 1$ ).

In concluding this section, it is important to point out that the conditions in Eqs. (1.6)–(1.10) are readily expressed using terminology more familiar in accelerator physics. In this regard, we introduce the (dimensionless) self-field perveance  $K_b$  and the unnormalized transverse emittance  $\epsilon$  defined by [130]

$$K_b = \frac{2N_b e_b^2}{\gamma_b^3 m_b \beta_b^2 c^2}, \quad (1.11)$$

and

$$\epsilon^2 = \frac{4}{\beta_b^2 c^2} [\langle x^2 + y^2 \rangle \langle v_x^2 + v_y^2 \rangle - \langle xv_x + yv_y \rangle^2]. \quad (1.12)$$

Here,  $N_b = \int dx dy d^3 p f_b$  is the number of beam particles per unit axial length,  $\langle x^2 + y^2 \rangle = R_b^2$  is the mean-square beam radius, and the unnormalized beam emittance  $\epsilon$  has dimensions of length. Then the conditions in Eqs. (1.7) and (1.8) corresponding to a *very-low-intensity, emittance-dominated* beam can be expressed in the equivalent forms

$$\frac{1}{2} K_b \ll \frac{\omega_{\beta\perp}^2 R_b^2}{V_b^2}, \quad (1.13)$$

and

$$K_b \ll \frac{1}{2} \frac{\epsilon^2}{R_b^2}, \quad (1.14)$$

where  $V_b = \beta_b c$ . On the other hand, for a *very-low-emittance, space-charge-dominated* beam, the conditions in Eqs. (1.9) and (1.10) can be expressed in the equivalent forms

$$\frac{1}{2} K_b \rightarrow \frac{\omega_{\beta\perp}^2 R_b^2}{V_b^2}, \quad (1.15)$$

and

$$K_b \gg \frac{1}{2} \frac{\epsilon^2}{R_b^2}. \quad (1.16)$$

A central focus of this treatise is to develop a theoretical formalism that describes self-consistently the nonlinear dynamics and collective processes in intense charged particle beams, spanning the entire range of beam intensities ( $K_b$ ) and emittances ( $\epsilon$ ) extending from the region of low beam intensity [Eqs. (1.13) and (1.14)] to the region of very high beam intensity [Eqs. (1.15) and (1.16)].

## 1.4 Outline of Contents

A major motivation for this treatise is to provide a unified theoretical treatment of a broad range of physical phenomena pertaining to the fundamental nonlinear dynamics and collective processes in intense charged particle beams, with particular emphasis on parameter regimes in which the self fields produced by the beam space charge and current play a significant role in determining the nonlinear evolution of the system. The subject matter is treated systematically from first principles, making extensive use of the theoretical techniques developed in the description of one-component nonneutral plasmas and electrically neutral plasmas. Chapter 2 begins with a description of the statistical frameworks for describing collective and discrete-particle interactions in intense charged particle beams based on the Vlasov-Maxwell equations, the macroscopic fluid-Maxwell equations, and the Klimontovich-Maxwell equations. Chapter 3 describes several fundamental properties of the single-particle motion in periodic

focusing field configurations, ranging from an alternating-gradient quadrupole field, to a periodic focusing solenoidal field, to a uniform focusing field (smooth-focusing approximation). For the case of an axially continuous beam with step-function density profile, the single-particle dynamics and envelope equations are analyzed in detail to illustrate the important influence of self-field effects at high beam current and charge density. In Chapters 4–8, we make use of the Vlasov-Maxwell equations to investigate the nonlinear dynamics and collective processes in intense charged particle beams. Such a kinetic model of intense beam propagation describes the self-consistent evolution of the beam distribution function  $f_b(\mathbf{x}, \mathbf{p}, t)$  in the six-dimensional phase space  $(\mathbf{x}, \mathbf{p})$  as the beam particles interact with the applied focusing field configuration and the average self-generated electric and magnetic fields,  $\mathbf{E}^s(\mathbf{x}, t)$  and  $\mathbf{B}^s(\mathbf{x}, t)$ . The Vlasov-Maxwell equations provide a complete theoretical framework for describing collective processes in intense charged particle beams. Quite remarkably, the analysis in Chapters 4–8 shows that many fundamental properties of beam propagation are accessible analytically, even in parameter regimes where the intense self-fields produced by the beam current and space charge play a major role in determining the detailed evolution of the system.

The specific topics covered in Chapters 4–8 include: the derivation of a three-dimensional nonlinear kinetic stability theorem (a *sufficient condition* for stability) based on global conservation constraints satisfied by the nonlinear Vlasov-Maxwell equations in Chapter 4; detailed studies of self-consistent beam equilibria determined from the steady-state ( $\partial/\partial t = 0$ ) Vlasov-Maxwell equations for intense beam propagation through a uniform focusing field configuration, and through periodic focusing quadrupole and solenoidal field configurations in Chapter 5; the derivation and application of global rate equations describing the nonlinear evolution of statistically-averaged beam properties in Chapter 6; the development of Hamiltonian averaging techniques for intense beam propagation through periodic focusing field configurations, which provides a rigorous framework for the smooth-focusing approximation in Chapter 7; and the investigation of collective oscillations and kinetic stability properties of an intense particle beam propagating through a uniform smooth focusing

field, using both analytical techniques based on the linearized Vlasov-Maxwell equations, and nonlinear perturbative simulation techniques (so-called  $\delta f$  method) in Chapter 8. In Chapter 9, neglecting the effects of heat flow, a macroscopic warm-fluid model is developed to describe intense beam equilibrium and collective stability properties. This macroscopic model, which is found to be remarkably robust, is applied to a detailed investigation of collective instabilities driven by pressure anisotropy ( $P_{\perp b} > P_{\parallel b}$ ) for perturbations about a warm-fluid waterbag equilibrium.

Finally, in Chapter 10, we conclude this treatise with a brief summary of several special topics on intense beam propagation not covered in Chapters 2–9. These topics include: beam propagation through background plasma, including self-pinch beam propagation, and the plasma lens effect; collisionless Landau damping (or growth) of collective excitations; investigations of multispecies effects, including the electron-ion two-stream instability in intense particle beams; production of beam halo particles produced by collective mode excitations in intense particle beams; and novel techniques for probing the beam microstate, such as the excitation and detection of beam echos.

An important conclusion from the analysis in Chapters 2–10 is that the Vlasov-Maxwell equations and the macroscopic fluid-Maxwell equations provide remarkably tractable and robust theoretical frameworks for describing the detailed nonlinear dynamics and collective processes in intense charged particle beams, including regimes where the intense self-fields produced by the beam space charge and current play a major role in determining the detailed evolution of the system.

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