

## Introduction

### The Theory of Condensed Matter

The science of condensed matter (thermodynamics, hydrodynamics, etc.) is in many ways much older than that of the atomic constituents. However, it is only in the last two or three decades that a systematic mathematical formulation of the many-body problem—with the  $10^{23}$  or so degrees of freedom needed to describe a macroscopic sample—has become developed so that the properties of the simplest classes of condensed matter can be related back quantitatively to the properties of the constituent atoms.

There are two fundamental classes of properties possessed by condensed matter which belong essentially to its many-body character and do not occur for the individual constituent degrees of freedom. One is the existence of propagation—the notion of a sound wave, the transport of electronic charge in metals, the propagation of light in insulators; the other is the occurrence of phase transitions by which the matter changes its fundamental symmetry—for classical systems melting and freezing, for quantum systems phenomena such as magnetism, superconductivity and superfluidity. Both classes of effects involve phenomena of long range, spreading over distances much greater than the effective range of the basic atomic forces which mediate them.

What is the nature of the mathematical construction which links the atomic to the condensed description of matter? Its formulation takes on many guises, but the essential feature is that, even when the individual atomic interactions may be treated, in some sense, as “weak,” the properties of the condensed system can only be treated correctly by taking them into account in infinite order. The most elementary example of such an infinite-order process leads to the occurrence of propagation through the “handing on” of excitation energy from one atom to the next. We shall see in Chaps. 1–3 that a natural formalism with which to set up the theory of the propagation phenomenon is the Green’s function

approach, which in a classical system reduces to the theory of correlation functions.

For a single degree of freedom the Green's function, or inverse differential operator, gives the amplitude of the degree of freedom at time  $t$ , given its amplitude at some previous time  $t'$ . This may refer either to a localized degree of freedom (e.g., a single atomic oscillator) or to a non-localized system (e.g., the amplitude of an electron wave function at position  $\mathbf{x}$  at time  $t$ , given that at position  $\mathbf{x}'$  at time  $t'$ ). The many-body effects are then embodied in the repeated emission of Huyghens wavelets as the electron propagates through the medium, giving rise to an infinite series of multiply scattered waves which sums to provide the Green's function for an electron interacting with the medium (which may consist of other electrons). In this way one can obtain the response of complex interacting systems to simple forms of excitation without having to find the full eigenvalue spectrum, a task which is generally neither practicable nor of physical interest. The relation between the Green's function formalism and scattering theory will be studied in Chaps. 4 and 5.

A major simplification which occurs for homogeneous many-body systems is that the low-lying excited states with energies near the ground state can often be simply described in terms of the resulting propagating modes. Because of their mode-like nature (with rather well-defined excitation energy  $\omega_{\mathbf{k}}$  as a function of the propagation wave-vector  $\mathbf{k}$ ) the quanta of these elementary excitations are referred to as quasiparticles. The Green's function approach to the theory of elementary excitations is developed in Chaps. 1-6 through the study of a series of specific examples drawn from solid state physics. We shall see how the Green's function determines the excitation spectrum through its analytic properties in the complex energy plane. We also show how the excitations produced by applied external fields can be formulated in terms of Green's functions, leading to general expressions for measured quantities such as electrical conductivities and magnetic susceptibilities. An important property of Green's functions is that they are related, via the fluctuation-dissipation theorem, to time correlation functions which determine scattering cross-sections and which also give the averages needed to discuss properties of the ground state of the system (or, at finite temperature, the thermal equilibrium state). These relationships are illustrated a number of times in the text.

The phase transition phenomenon can also be reached via the elementary excitation concept. For some quantum systems this may be studied at zero temperature by seeing how an excited state, of lower symmetry than the ground state, becomes degenerate with the ground state as the interaction strength is increased. This “softening” of the excitation energy will show up as a singularity of the Green’s function at the instability point of the system. In classical systems this singular behavior reduces to the Ornstein-Zernike theory of the two-particle correlation function. In Chaps. 7-10 we discuss the instability phenomenon in the context of the magnetic and superconducting instabilities of the interacting electron system. We also show how, once in the state of lower symmetry, the system acquires a new spectrum of elementary excitations which are no longer unstable.