

frequency of any variation applied to the plasma, $\omega_{gs} \gg \omega$. In the opposite case, when its cyclotron frequency is low, this particular species behaves as if the plasma would not contain a magnetic field. Because of the different particle masses, different plasma components may have a different magnetization behavior for a given variation frequency, ω .

As with the plasma frequency, there is a relation between the thermal velocity of a species and the cyclotron frequency of its particles

$$v_{\text{ths}} = \omega_{gs} r_{gs} \quad (1.7)$$

This equation defines the *gyroradius*, r_{gs} , of species s . This particular length is the radius of the circular orbit a particle performs in its motion around the magnetic field.

The gyroradius given above is actually the thermal gyroradius, because it is defined through the thermal velocity of the species. It is the average gyroradius of the particles of the particular species. Of course, each particle has its own gyroradius, depending on its velocity component perpendicular to the magnetic field. The gyroradius increases with velocity and also with mass or, better, it increases with particle energy. Energetic particles thus have large gyroradii.

Finally, we introduce one particular important quantity used in plasma physics, i.e., the ratio of thermal-to-magnetic energy density in the plasma, the so-called *plasma beta*

$$\beta = \frac{nk_B T}{B^2/2\mu_0} \quad (1.8)$$

This ratio tells us whether the plasma is dominated by the thermal pressure in the plasma or if the magnetic field dominates the dynamics of the plasma. Clearly, for $\beta > 1$ the former case is realized, and the magnetic field plays a relatively subordinate role, while in the opposite case, when $\beta < 1$, the magnetic field governs the dynamics of the plasma. The dynamics of the magnetospheric plasma is controlled by the geomagnetic field, while in the solar wind beta is large, and the dynamics of the solar wind plasma is dominated by the solar wind flow.

1.2. Particle Motions

Single particle motion in a plasma is naturally strongly distorted by the presence of all the other particles, the propagation of disturbances across a plasma, and a number of other effects. However, due to the Debye screening, the particles move approximately freely in a dilute collisionless and hot plasma for distances larger than one Debye length. One can assume that the small distortions of the particles caused by their participation in the *Debye screening* of the Coulomb fields of the other particles they pass along in their motion will in the average be small and will constitute only negligible wiggles around their collisionless orbits. This kind of wiggling in a more precise theory can be described by the *thermal fluctuations* of the particle density and velocity.

Within these assumptions it is possible to calculate the particle orbits. The particle orbits satisfy the single particle equation of motion in which all the collisional interactions with other particles and fields are neglected. Given external magnetic, \mathbf{B} , and electric fields, \mathbf{E} , this equation of motion reads

$$m_s \frac{d\mathbf{v}_s}{dt} = q_s (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) \quad (1.9)$$

The motion of the particles along the field lines is independent of the magnetic field and, in the absence of a parallel electric field component, $E_{\parallel} = 0$, the parallel particle velocity remains constant, $v_{\parallel} = \text{const}$.

The transverse particle motion can be split into a number of independent velocities if it is assumed that the gyromotion is sufficiently fast with respect to a bulk speed perpendicular to the magnetic field (see Chap. 2 of *Basic Space Plasma Physics*). Averaging over the circular *gyromotion*, the particle itself can be replaced by its *guiding center*, i.e., the center of its gyrocicle.

The velocity of the guiding center may be decomposed into a number of *particle drifts*. In a stationary perpendicular electric field the *Lorentz force* term in the above equation of motion tells that a simple transformation of the whole plasma into a coordinate system moving with the *convection* or $E \times B$ drift given in Eq. (1.2.19)

$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \quad (1.10)$$

cancels the electric field. In this co-moving system the particle motion is independent of \mathbf{E}_{\perp} , the perpendicular component of the electric field. It is force-free. Obviously, all particles independent of their mass or charge experience this drift motion, which is a mere result of the Lorentz transformation.

For time varying electric fields another drift arises, the so-called *polarization drift* given in Eq. (1.2.24)

$$\mathbf{v}_P = \frac{1}{\omega_{gs} B} \frac{d\mathbf{E}_{\perp}}{dt} \quad (1.11)$$

where the time derivative is understood as the total convective derivative. This drift depends on the mass and charge state of the species under consideration. Heavy particle drift faster than light particles. In addition, the directions of the drifts are opposite for opposite charges, leading to current generation. This drift is important for all low-frequency transverse plasma waves.

These drifts follow from a consideration of single-particle motions in electric and magnetic fields. As pointed out, plasmas do usually not behave like single particles. Only in rare cases, of which the *ring current* in the inner magnetosphere is an example (see Chap. 3 of the companion volume, *Basic Space Plasma Physics*), the motion of a single energetic particle mimics the motion of the entire energetic plasma component,

and the single particle drifts are useful tools for the description of the plasma dynamics. In all other cases one must refer to a *collective behavior* of the plasma which arises from the internal correlations between particles and fields even in the collisionless case. The plasma may then be considered not to consist of single particles but of particle fluids species. Each fluid can have its own density, bulk speed, pressure and temperature.

Such fluids when immersed into a magnetic field experience a *diamagnetic drift* which has been derived in Eq. (1.7.72). Obviously, this drift is a *collective effect* insofar as the collective particle pressure comes into play

$$\mathbf{v}_{dia,s} = \frac{\mathbf{B} \times \nabla_{\perp} p}{q_s n_s B^2} \quad (1.12)$$

Like the polarization drift, this bulk *pressure gradient drift* motion leads to currents, drift waves, may cause instability and nonlinear effects.

For completeness we mention that inhomogeneities and curvatures in the magnetic field generate additional drift motions in a plasma. In the present volume we will not make use of these, but refer the reader to the companion volume, *Basic Space Plasma Physics*, for reference.

1.3. Basic Kinetic Equations

Single particle effects, like the particle motion reviewed in the previous section, are often hidden in a plasma. In general, plasma dynamics cannot be described in such a simple way, but is determined by complicated correlations between particles and fields. The full set of basic equations of a plasma consists of the two *Maxwell equations*

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \quad (1.13)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1.14)$$

which must be completed by the two additional conditions, the absence of magnetic charges and Poisson's equation for the electric charge density, ρ

$$\nabla \cdot \mathbf{B} = 0 \quad (1.15)$$

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \quad (1.16)$$

The current and charge densities are defined as the sums over the current and charge densities of all species

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v}_s \quad (1.17)$$

$$\rho = \sum_s q_s n_s \quad (1.18)$$