Preface to This Edition

Equilibrium statistical mechanics, as created by Maxwell, Boltzmann, and especially Gibbs, had obvious potential for rigorous mathematical treatment. Some topics were really begging for analysis, like the infinite system limit, which is needed to understand the extensive property of thermodynamic quantities. But this was not generally understood in spite of early work by Onsager, Van Hove, Bogolyubov, Lee, Yang, and a few more. Only in the 1960's did a concerted effort of a group of young mathematical physicists turn to proving rigorous results in statistical mechanics. This was a highly successful enterprise, important results accumulated rapidly, and they were put together in the present book, first published in 1969.

Of course, progress has continued after that, covering new domains. Fortunately *Statistical mechanics: rigorous results* retains its basic usefulness, because only a small part of the contents has been superseded by new results, and much of the rest has not appeared again in book form. The reader will find further developments in the books of Ruelle [1], Israel [2], Bratteli and Robinson [3], Glimm and Jaffe [4], Sinai [5], and Simon [6]. From these, he or she will be able to assess the amazing vitality of equilibrium statistical mechanics. Unexpectedly, the mathematics underlying this subject have played a crucial role in the development of differentiable dynamics, operator theory, relativistic quantum mechanics, and renormalization group theory, among other things. One may thus be confident that the mathematics of equilibrium statistical mechanics will continue to play a major conceptual role in the future of science.

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DECEMBER 1998

[1] D. Ruelle. *Thermodynamic formalism*. Encyclopedia of Math. and its Appl., vol. 5, Addison-Wesley, Reading, Mass., 1978.

[2] R. Israel. *Convexity in the theory of lattice gases*. Princeton University Press, Princeton, NJ, 1979.

[3] O. Bratteli and D.W. Robinson. Operator algebras and quantum statistical mechanics. Springer, New York, 1979.

[4] J. Glimm and A. Jaffe. *Quantum physics: a functional integral point of view*. Springer, New York, 1981.

[5] Ia. G. Sinai. *Theory of phase transitions: rigorous results*. Pergamon, Oxford, 1982.

[6] B. Simon. *The statistical mechanics of lattice gases. I.* Princeton University Press, Princeton, NJ, 1993.

Special Preface

The *rigorous results* in this book largely originated in an unusual historic circumstance, where a small group of people applied the powerful methods of twentieth-century mathematics to a field that had lain fallow for several decades. The approach was remarkably successful, and so was the book.

The rigorous approach to Statistical Mechanics—luckily—goes far beyond the contents of this book, especially because it was realized that Relativistic Quantum Field Theory is in some sense a special case of classical Equilibrium Statistical Mechanics. The book—luckily—has aged well, and only a small part of its contents has been superseded by new results. Among the later results I shall only quote Asano's beautiful proof of the Lee-Yang circle theorem[1]. For other developments the reader is advised to consult the books of Ruelle[2], Israel[3], Bratteli and Robinson[4], Glimm and Jaffe[5], and Sinai[6]. From these, he or she will be able to assess the amazing vitality of Equilibrium Statistical Mechanics, and its unexpected mathematical relations with Differential Dynamics, Operator theory, Relativistic Quantum Mechanics, and Renormalization Group theory.

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OCTOBER 1988

[1] T. Asano, "Lee-Yang theory and the Griffiths inequality for the anisotropic Heisenberg ferromagnet," Phys. Rev. Letters *24*, 1409–1411 (1970). T. Asano, "Theorems on the partition functions of the Heisenberg ferromagnets," J. Phys. Soc. Japan *29*, 350–359 (1970). M. Suzuki and M. E. Fisher, "Zeros of the partition function for the Heisenberg, ferroelectric, and general Ising models," J. Math. Phys. *12*, 235–246 (1971). D. Ruelle, "Extension of the Lee-Yang circle theorem," Phys. Rev. Letters *26*, 303–304 (1971).

[2] D. Ruelle, *Thermodynamic formalism.* Encyclopedia of Math. and its appl., Vol. 5, Addison-Wesley, Reading, Mass., 1978.

[3] R. Israel, *Convexity in the theory of lattice gases,* Princeton University Press, Princeton, N. J., 1979.

[4] O. Bratteli and D. W. Robinson, *Operator algebras and quantum statistical mechanics,* Springer, New York, 1979.

[5] J. Glimm and A. Jaffe, *Quantum physics: a functional integral point of view,* Springer, New York, 1981.

[6] Ia. G. Sinai, *Theory of phase transitions: rigorous results,* Pergamon, Oxford, 1982.

Preface

Physics is a human attempt at understanding a certain class of natural phenomena, using scientific methods. We shall not try to define scientific methods in general, but note that they have been very successful and very diverse. Mathematical physics is one of these methods or approaches in which a mathematical structure is somehow associated with the structure of physical phenomena. The mathematical structure is then studied by mathematical techniques to yield new statements with physical relevance. Provided it yields physically relevant statements, no mathematical technique is to be judged too sophisticated or too trivial.

Applying the tools of mathematics to the investigation of the physical world may be a very rewarding intellectual experience. On the one hand, the knowledge which we have of physical phenomena hints at new mathematical theorems and at ways of providing them. On the other hand, mathematical analysis gives to the physical world a new structure and meaning. The knowledge of this structure and meaning constitutes an understanding of the "nature of things" as deep as we can hope to get.

Not every field of physics yields interesting mathematical physics. Luckily, we live in a period with many unsolved problems that are interesting and appear amenable to treatment. An exception to this statement may be relativistic quantum mechanics, largely because of "overgrazing," but there are also vast areas of *terra incognita.*

The present monograph is devoted to the study of a certain class of rigorous results in equilibrium statistical mechanics. These results concern the limit of an infinite system and the nature of phases and phase transitions. The emphasis is on general methods, and special models are essentially disregarded. (Exactly soluble models would deserve a book in their own right.)

The understanding of phase transitions remains one of the major unsolved problems of mathematical physics. The following chapters give an introduction to this problem, where some fundamental progress has taken place in recent years. After some generalities on ensembles (Chapter 1), we analyze the infinite system limit of thermodynamic functions for lattice systems (Chapter 2) and continuous systems (Chapter 3). We proceed with a detailed study of dilute systems (Chapter 4) and with a description of what is known on phase transitions (Chapter 5). We then discuss the nature of the states of physical systems with an invariance group (Chapter 6), and we analyze in particular the equilibrium states of infinite systems in statistical mechanics (Chapter 7). An appendix describes some mathematical tools used in Chapters 6 and 7, as these chapters are mathematically more sophisticated than the preceding ones. (The progress of mathematical physics could be significantly promoted, in the author's opinion, by the availability of the results of important mathematical theories in concise form and without proofs, in the spirit of Bourbaki's "Fascicules de Résultats." The appendix of this monograph is an attempt in that direction.) The logical relations between the chapters are roughly as follow:

$$\begin{array}{ccccc} 4 & \longrightarrow & 5 & & \\ \uparrow & & \uparrow & & \\ 1 & \longrightarrow & 2 & \longrightarrow & 3 \\ \downarrow & & \downarrow & & \\ 6 & \longrightarrow & 7 & & \end{array}$$

The people who have developed the subject matter of this monograph may be working in mathematics, physics, or chemistry departments. This diversity is one of the charms of the topic, but it creates a problem of language; the reader should remember this if he finds the text too mathematical, or too unmathematical.

The field of rigorous results in statistical mechanics extends beyond what is included in this volume (general results about equilibrium systems). A few pieces of work that are not treated include the proof of stability of quantum Coulomb systems by Dyson and Lenard[1], the study of the two-phase region for the Ising model by Minlos and Sinai[1], and the analysis of the time evolution of infinite classical systems by Lanford[1].

I am much indebted to G. Gallavotti, C. Gruber, O. Lanford, S. Miracle-Sole, and A. S. Wightman for suggesting many corrections and improvements to the manuscript of this book.

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